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Analytical Model for Compact Star in a Buchdahl Spacetime Consistent with Observational Data

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Abstract

A method is developed to obtain solutions of Einstein field equations for anisotropic charged spheres. This procedure needs to choose a linear relationship between energy density and radial pressure and a metric function proposed for Buchdahl (1959). A new class of solution is obtained and subjected to several physical analyses for realistic models of compact stars. The new solutions in this research are physically reasonable, well-behaved in the interior of the star, which indicates that these new models satisfies important physical conditions as the measure of anisotropy and matching. The models are consistent with the upper limit on the mass of compact stars for PSR J1823-3021G, PSR J1748-2446an and PSR J1518+4904.

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Anisotropy; Compact Star; Einstein-Maxwell System; Linear Equation of State; Metric Potential.

Introduction

The phenomena of supernovae stars giving birth to strange stars through gravitational collapse has motivated a number of researchers to explore the geometry of stellar inner portions.[1-2] In general relativity the Einstein field equations are useful in examining the physical characteristics and gravitating behaviours of some known stellar objects such as the star remnants.[3-5] The essence of these models show that the field equations are useful and applied as tools to provide results with astrophysical significance.[6-18]

One of the groundbreaking developments in the theory of general relativity was made by Schwarzschild[3] who derived the first solution to Einstein's field equations. This solution has been crucial in understanding the behavior of massive objects and their interaction with gravity and has allowed to obtain Einstein's original cosmological solutions for a uniform distribution of fluid.

Modeling compact stellar objects has become a popular and important endeavor to explore various characteristics including their mass, charge, structure and stability.[19] Some reasonable physical stellar models can be proposed with various state equations as the linear equation of state,[20-26] quadratic equation of state,[27-30] polytropic equation of state[31-32] and Van der Waals equation of state.[33-34]

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The study of anisotropy pressure in stellar objects in the presence of strong gravitational fields is a topic of fundamental importance for many researchers in astrophysics. Sokolov[18] states that phase transitions are determinants in the evolution of neutron stars. The presence of an electrical field is also a cause of anisotropy.[19] Bowers and Liang6 indicate that the presence of anisotropy can modify the structure of compact objects. Herrera[11] concludes that the pressure anisotropy influences matter stability due to the appearance of radial forces of different sign in the stellar interior causing a disturbance in the system balance. Thirukkanesh and Ragel[35] state that anisotropy influences the structure and some physical parameters of compact stars such as mass and compactness. Moreover, there exist a number of research studies that have come up with anisotropic models (Takisa and Maharaj,[31] Thirukkanesh and Ragel,[32] Malaver,[33-34] Thirukkanesh and Ragel,[35] Mak and Harko,[36] Malaver and Iyer).[37]

Malaver and researches[38] have discussed the effect of electromagnetic fields on compact stellar bodies in a Buchdahl space time. Malaver, Iyer, and Khan[39] have determined some physical characteristic in the framework of Einstein-Gauss-Bonnet gravity for compact stellar objects with the metric potential proposed by Buchdahl. Iyer[40-41] has recently published many papers as well as presentations on the importance of Rank-n tensor time quantifying gravity in quantum states with gravity and tensor time metrics. In these studies, the gradation of time tensors from rank-6 to rank-1 vectors in spacetime presents a novel approach to unifying General Relativity (GR) and Quantum Relativity (QR).

The main purpose of this study is to obtain a new variety of explicit solutions of Einstein field equations with a metric function proposed by Buchdahl[42] and considering the existence of pressure anisotropy. In section 2 are shown the field equations and boundary conditions that describe the gravitational behavior of the astrophysical objects and the solutions for the Einstein field equations are given by in Section 3. In Section 4, are discussed the physical conditions that must have a charged star and the physical analysis of particular cases are given in Section 5. Finally in Section 6, we conclude that the proposed model describes a charged stable star and that the matter variables can contribute to the study of stellar structure.

Einstein Field Equations

We considered a distribution of matter with spherical symmetry whose stress tensor is locally anisotropic. The metric in a star in Schwarzschild coordinates will be described by the simple form

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad ...(1)$$

with λ and v functions of r only, as in the case of any static and spherical symmetric distribution of matter. The Einstein-Maxwell field equations given by

$$T_{00} = -\rho - \frac{1}{2}E^2 \quad ...(2)$$

$$T_{11} = p_r - \frac{1}{2}E^2 \quad ...(3)$$

$$T_{22} = T_{33} = p_t + \frac{1}{2}E^2 \quad ...(4)$$

The quantities ρ, P_r, p_t and E refer to as energy density, radial pressure, tangential pressure and electric field, respectively. The basic field equations (2)-(4) are transformed to find the solution to the Einstein field equations with the transformations, $x=cr^2$, $Z(x)=e^{-2\lambda(r)}$ and $A^2 y^2(x)=e^{-2\nu(r)}$ with arbitrary constants A and c>0, suggested by Durgapal and Bannerji.[43] The metric (1) can be expressed as

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad ...(5)$$

and Einstein field equations are written as follows

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \quad ...(6)$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \quad ...(7)$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \quad ...(8)$$

$$p_t = p_r + \Delta \quad ...(9)$$

$$\frac{\Delta}{c} = 4xZ\frac{\ddot{y}}{y} + \dot{Z}\left(1 + 2x\frac{\dot{y}}{y}\right) + \frac{1-Z}{x} - \frac{E^2}{c} \quad ...(10)$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \quad ...(11)$$

σ is the charge density and dots in system of equations (6)-(11) stand for derivatives with respect to x. With the transformations of Durgapal and Bannerji,[43] the mass within a radius r for the realistic stellar body is given by for

$$M(x) = \frac{1}{4c^{3/2}}\int_0^x \sqrt{w}\rho(w)dw \qquad ...(12)$$

Matching of the exterior and interior of the compact object at the boundary (r = R) is done by comparing the line element (1) with Reissner-Nordstrom exterior spacetime.

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2 + \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2+\sin^2\theta d\varphi^2) \qquad ...(13)$$

In this paper, we assume the following lineal equation of state

$$p_r = \frac{1}{3}\rho \qquad ...(14)$$

Materials and Methods

To be able to solve the system (6)-(11) we have taken the metric potential Z*(x)* of Buchdahl[42] and for the electric field the proposal of Lighuda and researches,[44] respectively

$$Z(x) = \frac{K+x}{K(1+x)} \qquad ...(15)$$

$$\frac{E^2}{2c} = \frac{ax}{1+ax^2} \qquad ...(16)$$

where K is a parameter related to the geometry of the star and a is a real constant. The metric potential Z*(x)* is continuous at the centre of the star and well behaved in the interior of the star. The electric field is finite at the center of the star and remains continuous in the interior.

Substituting (15) and (16) in (6) we obtain

$$\rho = c\left[\frac{(K-1)(1+x)+2(K-1)}{K(1+x)^2} - \frac{ax}{1+ax^2}\right] \qquad ...(17)$$

we have for the radial pressure

$$P_r = \frac{1}{3}c\left[\frac{(K-1)(1+x)+2(K-1)}{K(1+x)^2} - \frac{ax}{1+ax^2}\right] \qquad ...(18)$$

Using (17) in (12), the expression of the mass function is

$$M(x) = -\frac{\sqrt{x}}{2\sqrt{c}}\left[\frac{K+x}{K(x+1)}\right] + \frac{\sqrt{2}\left(1/a\right)^{1/4}}{8\sqrt{c}}\left[\begin{array}{l}\arctan\left(\frac{\sqrt{2x}}{\left(1/a\right)^{1/4}}+1\right)+\arctan\left(\frac{\sqrt{2x}}{\left(1/a\right)^{1/4}}-1\right)+ \\ \frac{1}{2}\ln\left(\frac{x+\sqrt{2x}\left(1/a\right)^{1/4}+\sqrt{1/a}}{x-\sqrt{2x}\left(1/a\right)^{1/4}+\sqrt{1/a}}\right)\end{array}\right] \qquad ...(19)$$

With (15) and (16) in eq. (11), the charge density is

$$\sigma^2 = \frac{2ac^2(K+x)\left[2+\sqrt{2}+\left(2-\sqrt{2}\right)ax^2\right]^2}{K(1+x)\left(1+ax^2\right)^3} \qquad ...(20)$$

With (15), (16) and (18), the eq. (7) becomes

$$\frac{4(K+x)\dot{y}}{K(1+x)y} = \frac{K(1+x)-(K+x)}{Kx(1+x)} + \frac{(K-1)(1+x)+2(K-1)}{3cK(1+x)^2} - \frac{ax}{3c\left(1+ax^2\right)} - \frac{ax}{1+ax^2} \qquad ...(21)$$

Integrating (21), we obtain

$$y(x) = c_1(1+x)^{1/6}(K+x)^{A*}\left(1+ax^2\right)^B e^{C\arctan\left(x\sqrt{a}\right)} \qquad ...(22)$$

where for convenience

$$A_* = -\frac{aK^2-2K+3}{6\left(aK^2+1\right)} \qquad ...(23)$$

$$B = -\frac{K(aK+1)}{6\left(aK^2+1\right)} \qquad ...(24)$$

$$C = \frac{\sqrt{a}K(K-1)}{3\left(ak^2+1\right)} \qquad ...(25)$$

The metric functions can be written as

$$e^{2\lambda(r)} = \frac{K(1+x)}{K+x} \qquad ...(26)$$

$$e^{2\nu(r)} = A^2c_1^2(1+x)^{1/3}(K+x)^{2A}\left(1+ax^2\right)^{2B}e^{2C\arctan\left(x\sqrt{a}\right)} \qquad ...(27)$$

The anisotropy factor Δ is given by for

$$\Delta = \frac{4xc(K+x)}{K(1+x)}\left[\begin{array}{l}\frac{A_*^2-A_*}{(K+x)^2}+\frac{A_*}{3(K+x)(1+x)}+\frac{4A_*Bax+2A_*C\sqrt{a}}{(K+x)\left(1+ax^2\right)}-\frac{5}{36(1+x)^2} \\ +\frac{2Bax+C\sqrt{a}}{3(1+x)\left(1+ax^2\right)}+\frac{4\left(B^2-B\right)a^2x^2}{\left(1+ax^2\right)^2}+\frac{2Ba}{1+ax^2}+\frac{2(2B-1)Ca^{3/2}x}{\left(1+ax^2\right)^2}+\frac{C^2a}{\left(1+ax^2\right)^2}\end{array}\right]$$
$$\frac{(1-K)c}{K(1+x)^2}\left[1+2x\left(\frac{A_*}{K+x}+\frac{1}{6(1+x)}+\frac{2Bax+C\sqrt{a}}{1+ax^2}\right)\right]+\frac{c(K-1)}{K(1+x)}-\frac{2acx}{1+ax^2} \qquad ...(28)$$

Physical Requirements

Any physically acceptable solution must satisfy the following conditions,[32,45]

(i) The gravitational potentials $e^{2\lambda}$ and $e^{2\nu}$ are functions that take finite and positive values along the radial coordinate and are continuous throughout the stellar interior.

(ii) The energy density ρ decreases continuously from the centre r=0 and becomes zero at the surface r=R.

(iii) The radial pressure P_r must be finite at the centre and it must vanish at the surface of the sphere.

(iv) The radial pressure and density gradients $dP_r/dr \leq 0$ and $d\rho/dr \leq 0$ for $0 \leq r \leq R$.

(v) The anisotropy is zero at the center r=0, i.e. Δ(r=0) =0.

(vi) Any physically acceptable model must satisfy the causality condition, that is, for the radial sound speed $v_{sr}^2 = \frac{dP_r}{d\rho}$, we should have $0 \le v_{sr}^2 \le 1$

(vii) At the surface of the star the charged interior solution should be matched with the Reissner–Nordström exterior spacetime (13).

Results and Discussion

For the new solutions, the gravitational potentials $e^{2\lambda}$ and $e^{2\nu}$ have finite values and are continuous throughout the stellar interior, this is in agreement with the result by Sunzu, Maharaj and Ray.[26] At the center $e^{2\lambda(0)} = 1$ and $e^{2\nu(0)} = A^2 c_1^2 K^{2A} e^{2C}$. We show that in r=0,$(e^{2\lambda(r)})'_{r=0} = (e^{2\nu(r)})'_{r=0} = 0$ and this makes is possible to verify that the gravitational potentials should be finite at the center and avoid the singularities within the stellar interior.

The energy density ρ and radial pressure P_r are decreasing functions with maximum values at the centre of the star. In the center $\rho(r=0) = \frac{3c(K-1)}{K}$ and $P_r(r=0) = \frac{c(K-1)}{K}$, therefore the energy density will be non-negative in r=0 and $P_r(r=0) > 0$. In the surface of the star $P_r(r=R) = 0$ and for the second fundamental form we have

$$\frac{(K-1)(1+cR^2)+2(K-1)}{K(1+cR^2)^2} - \frac{acR^2}{1+acR^4} = 0 \qquad ...(29)$$

Gradients $\frac{d\rho}{dr}$ and $\frac{dP_r}{dr}$ acquire negative values with the radial parameter. For $0 \le r \le R$

$$\frac{d\rho}{dr} = \frac{2(K-1)c^2 r}{K(1+cr^2)^2} - \frac{4((K-1)(1+cr^2)+2K-2)c^2 r}{K(1+cr^2)^3} - \frac{2ac^2 r}{ac^2 r^4+1} + \frac{4a^2c^4r^5}{(1+ac^2r^4)^2} < 0 \qquad ...(30)$$

$$\frac{dP_r}{dr} = \frac{2(K-1)c^2 r}{3K(1+cr^2)^2} - \frac{4((K-1)(1+cr^2)+2K-2)c^2 r}{3K(1+cr^2)^3} - \frac{2ac^2 r}{3ac^2 r^4+1} + \frac{4a^2c^4r^5}{3(1+ac^2r^4)^2} < 0 \qquad ...(31)$$

According equations (30) and (31) the pressure and density diminish in the stellar interior and vanishes on the surface of the star. The solution for r=R must match the Reissner–Nordström exterior space–time as:

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2 + \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2\left(d\theta^2+\sin^2 d\varphi^2\right)$$

and therefore, the continuity of e^{ν} and e^{λ} across the boundary r=R is

$$e^{2\nu} = e^{-2\lambda} = 1 - \frac{2M}{R} + \frac{Q^2}{R^2} \qquad ...(32)$$

Then for the matching conditions, we obtain:

$$\frac{2M}{R} = \frac{(K-1)cR^2 + 2ac^2KR^4 + (3K-1)ac^3R^6}{K(1+cR^2)(1+ac^2R^4)} \qquad ...(33)$$

Table 1 contains the values of a, K and masses for different stars in r=R

Table 1: Values of K, a and M(MΘ) in r=R

K	a	M(MΘ)
6	0.0002	2.71
8	0.0002	2.87
10	0.0002	2.96

MΘ = sun's mass

The figures 1,2,3,4,5,6,7,8 and 9 represent the plots of $e^{2\lambda}$, $e^{2\nu}$, ρ, P_r, $\frac{d\rho}{dr}$, $\frac{dP_r}{dr}$, M, σ^2 and Δ with the radial coordinate. For all the plots c=1.

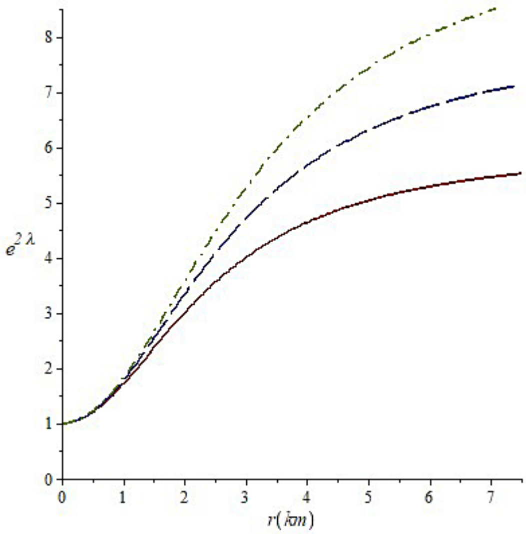

Fig. 1: Metric potential $e^{2\lambda}$ versus the stellar radius for K=6 (solid line), K=8 (long- dash line) and K=10 (dash-dot line) with a=0.0002.

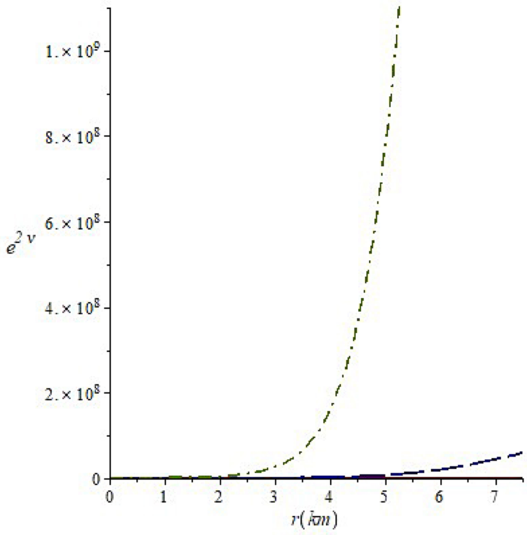

Fig. 2: Metric potential $e^{2\lambda}$ versus the stellar radius for K=6 (solid line), K=8 (long- dash line) and K=10 (dash-dot line) with a=0.0002.

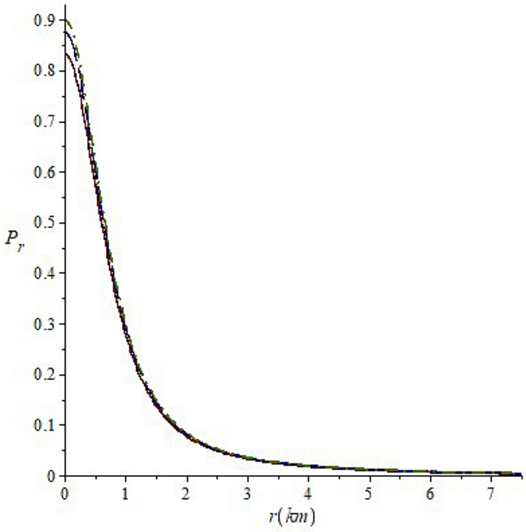

Fig. 4: Radial pressure P_r versus the stellar radius for K=6 (solid line), K=8 (long- dash line) and K=10 (dash-dot line) with a=0.0002.

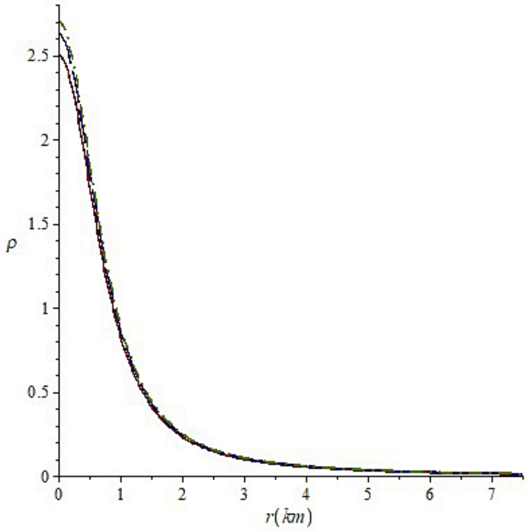

Fig. 3: Energy density ρ versus the stellar radius for K=6 (solid line), K=8 (long- dash line) and K=10 (dash-dot line) with a=0.0002.

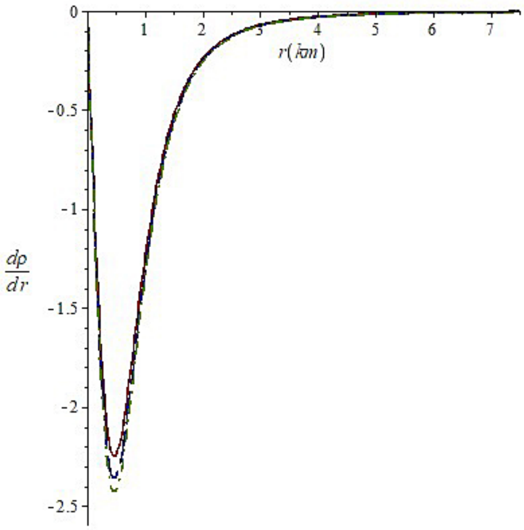

Fig. 5: Energy density gradient versus the stellar radius for K=6 (solid line), K=8 (long-dash line) and K=10 (dash-dot line) with a=0.0002.

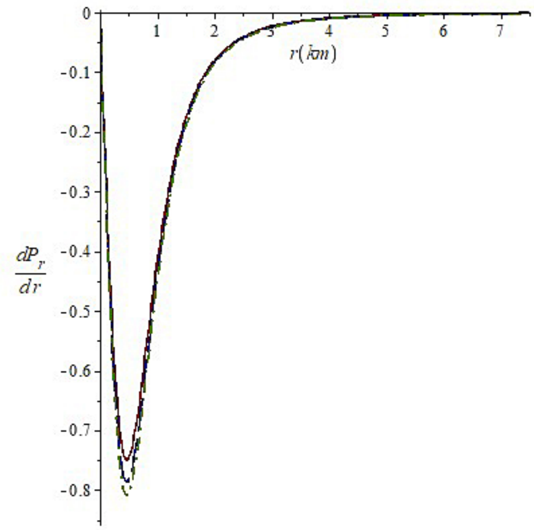

Fig. 6: Radial pressure gradient versus the stellar radius for K=6 (solid line), K=8 (long-dash line) and K=10 (dash-dot line) with a=0.0002.

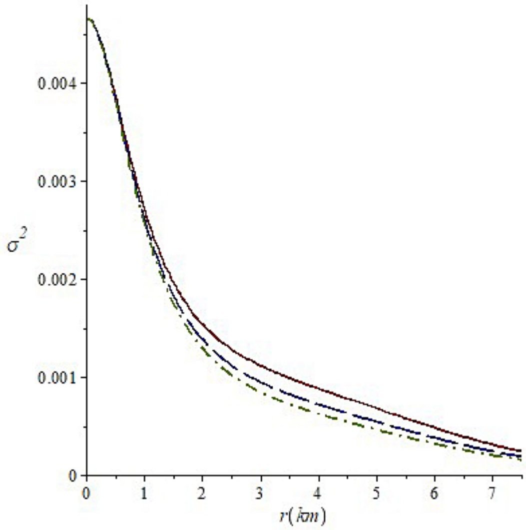

Fig. 7: Mass function versus the stellar radius for K=6 (solid line), K=8 (long- dash line) and K=10 (dash-dot line) with a=0.0002.

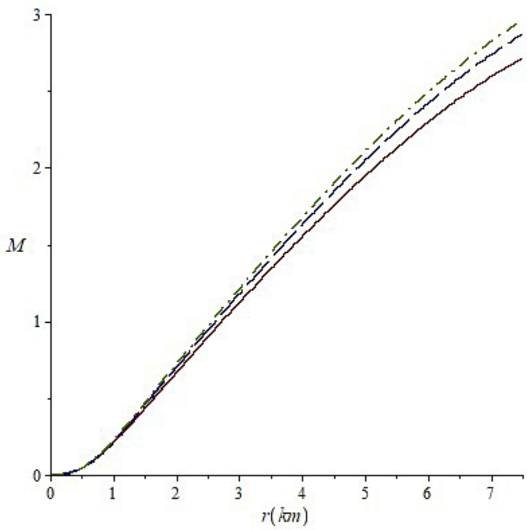

Fig. 8: Charge density σ^2 versus the stellar radius for K=6 (solid line), K=8 (long- dash line) and K=10 (dash-dot line) with a=0.0002.

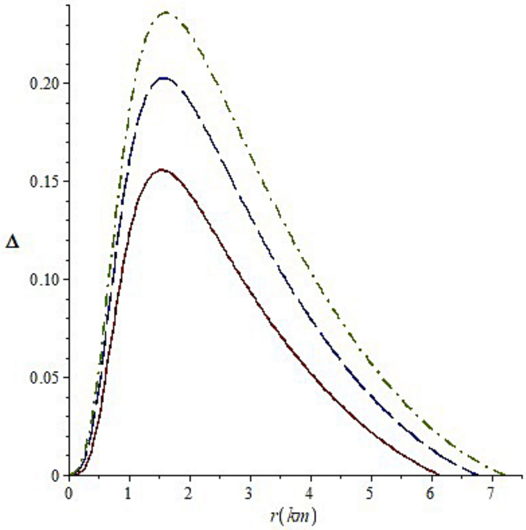

Fig. 9: Anisotropy versus the stellar radius for K=6 (solid line), K=8 (long- dash line) and K=10 (dash-dot line) with a=0.0002.

For different values of parameter K, the gravitational potentials $e^{2\lambda}$ (Figure 1) and $e^{2\nu}$ (Figure 2) are monotonically increasing function with the radial distance, continuous throughout the stellar interior and takes higher values when K is increased. The energy density from Figure 3, is observed to be maximum at the star core. It decreases as the radial distance increases for all the values of K.

This indicates that the model is physically realistic as we expect the maximum value of energy at the centre as observed by Sunzu, Maharaj and Ray.[26] The radial pressure is also a decreasing function with radial coordinate with the maximum at the centre as a noted in Figure 4. The radial variation of energy density gradient has been shown in Figure 5, in which it is observed that $\frac{d\rho}{dr} < 0$ in all the cases studied. In Figure 6 is also shown that the profile of $\frac{dP_r}{dr}$ indicates that the radial pressure gradient is negative inside the star. In Figure 7, the mass of the stellar body increases monotonically from the centre to the surface for different values of K. It is also physically realistic for well behaved models.[24] The charge density is a continuously decreasing function as noted in Figure 8. The measure of pressure anisotropy Δ in Figure 9 shows that it is finite, regular, continuous, and increasing from the core of stellar object, reaches a maximum and then decreases near the surface. We can also note that Δ admits higher values with a growth of K.

We can compare the values calculated for the mass function with observational data of some astrophysical objects such as for PSR J1823-3021G, PSR J1748-2446an and PSR J1518+4904.[46-48] The values of the stellar masses for these compact stars are tabulated in Table 2.

Table 2: The approximate values of the masses for the compact stars

Compact Star	Masses M(MΘ)
J1823-3021G	2.65
J1518+4904	2.72
J1748-2446an	2.97

The recently discovered pulsar PSR J1823 3021G is known to be part of a binary system and has the potential of being one of the most massive known pulsars.[47] The same is noted with the binary pulsar PSR J1748-2446an which is a massive system that exceeds 2MΘ.[48] Iyer[40-41] has outlined a method to quantize the gravitational field by examining the gradation of rank tensors within a metric wavefunction framework. Astrophysical regions would demonstrate a rank2 tensor, analogous to Schwarzschild metrics, by analyzing Einstein's Field Equations and the Schwarzschild metrics to explain the gravitational interaction in terms of spacetime curvature. To validate theoretical framework of rank tensor gradation and metric wave functionality, many experimental approaches have been advanced to proposals,[48-60] including high-energy particle collisions, gravitational wave observations, quantum entanglement experiments, astrophysical observations, and laboratory simulations.

Conclusion

This study included a choice of generalized metric function which has regained some choices made by previous researchers.[39] Moreover, the developed model was observed to be regular. That is, the potentials are above zero at the centre of a star showing that the model is regular. The proposed models can be compared with the pulsars PSR J1823-3021G, PSR J1748-2446an and PSR J1518+4904[43-44] and well behaved. Quantifying gravity in quantum states with gravity and tensor time metrics presents a novel approach to unifying General Relativity and Quantum Relativity. Astrophysical regions would demonstrate a rank2 tensor using Einstein's Field Equations and the Schwarzschild metrics to explain the gravitational interaction in terms of spacetime curvature. Many experimental approaches such as high-energy particle collisions, gravitational wave observations, quantum entanglement experiments, astrophysical observations, and laboratory simulations have promising advances to find signatures of the quantum interior with astro exterior of these compact stars, especially pulsars.[50-60]

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Conflict of Interest

The authors declares that there is no conflict of interest regarding the publication of this article.

Data Availability Statement

This statement does not apply to this article.

Ethics Statement

This research did not involve human participants, animal subjects, or any material that requires ethical approval.

Informed Consent Statement

This study did not involve human participants, and therefore, informed consent was not required.

Author Contributions

- **M.M (Manuel Malaver):** Supervision and Writing – Original Draft.
- **R.I (Rajan Iyer):** Review & Editing

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