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Study of Coupling Constants of Gravitational Waves using String Theory

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Abstract

In this paper, we study the coupling constants of quantized gravitational waves during its scattering and absorption by the black hole in string theory. In string theory, the quantized gravitational waves are closed strings of Planck size which are usually graviton particles in quantum gravity having spin 2. The coupling constants calculated in this paper in terms of winding number of the closed string have been found to form a particular sequence and have shown that they are cyclic vectors. We have calculated various sequences of the coupling constants of gravitational waves which come under the area of metric spaces. We have used the concepts of both T-duality and S-duality in order to explain both scattering and absorption of gravitational waves by the black hole.

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Introduction

Finding out the coupling constants of quantized gravitational waves in string theory are interesting problem of theoretical physics. Gravitational waves are ripples in space-time.[1,2,3,4] Several phenomena regarding the absorption and scattering of gravitational waves by the black hole are discussed in refs.[5–26] Various explanations about quantized gravitational waves in quantum gravity and loop quantum gravity are given in refs.[27–36] The quantized wavelength, time period and energy of the gravitational wave can be written as $\lambda_n=2\pi nL_P$, $P_n=2\pi nT_P$, $E_n= 1/n\ E_P$, $n=1,2,3\ldots$ where L_P, T_P and E_P are Planck length, Planck time and Planck energy respectively, and $E_P=M_P c^2$ in which M_P is defined as Planck mass.[34,35,36] Quantized gravitational waves are gravitons having spin 2 . Graviton is considered to be quantized form of the gravitational wave to be studied in quantum gravity and is a spin-2 particle.[37–47] Absorption of gravitational waves by the black hole are discussed in references.[48,49,50]

In quantum gravity, a graviton is considered as a point particle having zero size with spin 2.[37–47]

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Whereas, in string theory, particles are considered as strings which are extended objects having one dimension and hence a graviton[51] is considered as a closed string with spin-2 having Planck size. In both the cases they have spin 2. In string theory, gravitational waves are closed loops of energy each having spin 2. The energy of closed string depends upon its winding number i.e. higher the winding number, higher the energy and during scattering of gravitational waves or absorption of gravitational wave there is change in winding number of closed string which corresponds to the change in energy of closed string. String theory can be explained with strings and branes.[51,52,53,54,55,56] As given in reference 53, a string is a one-dimensional brane whereas a point particle is zero-dimensional brane and two-dimensional brane is called as membrane. We are living in three-dimensional brane i.e. all the bigger things we see around us lie in a three-dimensional brane. This means the visible universe with the naked eye is three-dimensional brane. This also means that we are living in the world in which 6 dimensions are compactified out of 9 dimensions in 10-dimensional string theory. In string theory, we have universal gravitational constant $G=l_p^2$ but the coupling constant g is a dimensionless quantity, so we have $l_p \sim g l_s$.Here, l_s is length of closed string having one winding number around one dimensional circle of radius R in which 25 dimensions are compactified to one dimensional closed string having circumference 2πR in 26 dimensional bosonic string theory. According to T-duality in theoretical physics, the theories describing the strings propagating in a circle of radius R in space-time is equivalent to the theories the strings propagating in a circle of radius $1/R$ in space-time.[57,58,59,60] In particle physics and quantum gravity, elementary particles are considered as point particles possessing certain coupling constants. But in string theory, all elementary particles are either one dimensional open string or one-dimensional closed string in which 25 dimensions in 26-dimensional string theory are compactified to one dimension or 9 dimensions in 10-dimensional string theory are compactified to one dimension and their coupling constants are different from those of point particles. In other words, an elementary particle is a single harmonic oscillator whereas in string theory, a string consists of infinite number of harmonic oscillators. In reference 60, T-duality has been explained in 10-dimensional string theories which is compactified in order to obtain theories with lower number of space-time dimensions d and one of the simplest compactifications are toroidal compactification which possess internal manifold as a n-dimensional torus T^n (n=10-d) leaving supercharges unbroken. Using T-duality, we can easily measure the winding number represented by the function as Fourier series$\varphi(\theta)=wR\theta+x+\sum_{(n\neq 0)} c_n e^{in\theta}$ where w is winding number of the closed string curved around the circle of radius R. The closed strings are wind around the circle of radius R once, twice, thrice or many times. It can also be unwind around the circle and hence a closed string has a certain winding number w. Since, quantized gravitational waves are closed strings in string theory, during the interaction of gravitational waves with the black hole, there is change in the winding number of the quantized gravitational wave. The winding numbers of the closed strings can be w = 0, 1, 2, 3…. If the turns are counterclockwise, then the winding number is positive whereas if the turns are clockwise, we have negative winding numbers. Also, the we can express the Hamiltonian of the string as: $H=wR/l_s^2)+(nl_s^2)/R+\sum_n|\dot{c}_n|^2+n^2|c_n|^2$, where w is winding number of the string curving around a circle of radius R and n/R represents the momentum of the closed string around the circle of radius $1/R$ along with this c_n is time dependent. In string theory, the coupling constants of the string is dependent upon the string's oscillation modes called as dilaton whose exchange is also the exchange of large coupling constants with small coupling constants and this symmetry is known as S-duality. According to S-duality, we have a theory with weak coupling constant same as the theory with strong coupling constant where we cannot understand strong coupling by using perturbation theory, but we can understand weak coupling by using perturbation theory. Hence, using S-duality,[60,61,62,63] understanding of string theory having strong coupling is equivalent to understand string theory having weak coupling. In this paper, we have utilized closed string winding numbers to compute the coupling constants of gravitational waves. The reason behind this is that the energy of closed string corresponds to winding number of closed strings. Since different coupling constants of gravitational waves as closed strings have different energies determine the dependence of coupling constant of gravitational wave on winding number of the closed string. As given by reference

63, the strong coupling regime in the quantum equivalence of two perturbatively distinct theories A and B, of A has mapping with the weak coupling regime of B. The perturbative excitations of A is to be mapped with the non-perturbative excitations of the dual theory and vice-versa. In this paper, we have utilized closed string winding number to compute the coupling constants of gravitational waves. The reason behind this is that the energy of closed string corresponds to winding number of closed strings. Since different coupling constants of gravitational waves as closed strings have different energies which determine the dependence of coupling constant of gravitational wave on winding number of the closed string. Our work regarding scattering and absorption of gravitational waves by the black hole can also be explained using superstring theory.[52,55,63]

In section 2 and section 3 of this paper, we discuss the change in winding number of the closed string during the scattering and absorption of gravitational waves by the black hole in string theory. In section 4, coupling constants of gravitational waves in string theory in terms of winding number of closed string with cyclic vectors and metric spaces using S-duality and T-duality in string theory are calculated. Finally, we present our conclusions in section5.

Scattering of Gravitational Waves by the Black Hole in String Theory

In the scattering of gravitational waves with a black hole, three cases arise. In the first case, the gravitational wave will scatter with the winding number lesser than the winding number initially it has. In the second case, it will scatter with winding number greater than initially it has. In third case, it will scatter with winding number equal to the winding number initially it has. Considering quantized gravitational waves as closed strings in string theory, from the equation given by:

$$l_p=g_1\, w_1\, l_s=g_2\, w_2\, l_s, \quad ...(1)$$

we will get the coupling constant of the closed string incident on the black hole as:

$$g_1=l_p/w_1\, l_s\,. \quad ...(2)$$

The gravitational wave will return back after collision with the black hole possessing winding number lesser than, greater than or same as the winding number initially it has, so the coupling constant of the closed string scattered by the black hole is given below as:

$$g_2 = \frac{l_p}{w_2 l_s}. \quad ...(3)$$

(a) In the first case, when the gravitational wave is scattered with winding number lesser than initially it has, then we have

$$w_2<w_1. \quad ...(4)$$

In this case, the difference in the winding number between incident gravitational wave and scattered gravitational wave as closed strings in string theory is given by,

$$\Delta w = w_1 - w_2 = \left(\frac{1}{g_1} - \frac{1}{g_2}\right)\frac{l_p}{l_s}. \quad ...(5)$$

In this case, the winding number of the closed string is decreased. The change in Hamiltonian energy of the closed string can be given as:

$$\delta H = \Delta w R/l_s^2 + \Delta\, n l_s^2/R. \quad ...(6)$$

Also, we have $\Delta w=\Delta n$using T- duality of string theory i.e. $w\sim n$in equation (6) and time-dependent term is cancelled out since the time-dependent term will be same for both incident and the scattered gravitational waves by the black hole. So, the Hamiltonian energy difference for the scattered gravitational waves for this case can be given using equations (5) and (6):

$$\delta H = \left(\frac{1}{g_1} - \frac{1}{g_2}\right)(R/l_s^2 + l_s^2/R)\frac{l_p}{l_s}. \quad ...(7)$$

(b) In the second case, when the gravitational wave is scattered with winding number greater than initially it has, then we have

$$w_2>w_1. \quad ...(8)$$

In this case, the difference in the winding number between incident gravitational wave and scattered gravitational wave as closed strings in string theory is given by,

$$\Delta w = w_2 - w_1 = \left(\frac{1}{g_2} - \frac{1}{g_1}\right)\frac{l_p}{l_s}. \quad ...(9)$$

In this case, since the winding number of the closed string is increased. Using T- duality of string theory i.e. $w\sim n$ in equation (6), the Hamiltonian energy difference for the scattered gravitational waves for this case can be given using equations (6) and (9):

$$\delta H = \left(\frac{1}{g_2} - \frac{1}{g_1}\right)(R/l_s^2 + l_s^2/R)\frac{l_p}{l_s}. \quad ...(10)$$

(c) In the third case, when the gravitational wave is scattered with the same winding number initially it has, we have

$$w_1=w_2=w. \quad ...(11)$$

In this case, the difference in the winding number between incident gravitational wave and scattered gravitational wave is given by,

$$\Delta w = 0 = \left(\frac{1}{g_2} - \frac{1}{g_1}\right)\frac{l_p}{l_s}. \quad ...(12)$$

Equation (12) gives us the value of the coupling constant $g_2=g_1=g$of the quantized form of gravitational wave as closed string which is scattered by the black hole. In this case, since the winding number doesn't change, the Hamiltonian of the string remains unchanged. Here also using T- duality of string theory i.e. w~n in equation (6), the Hamiltonian energy difference for the scattered gravitational waves for this case using equations (6) and (12) will be δH=0.

Absorption of Gravitational Waves

Like scattering, during absorption of gravitational waves, winding number of the incident gravitational wave as closed string becomes zero after its absorption by the black hole. After absorption of gravitational wave by the black hole, the winding number will be:

$$w_2=0. \quad ...(13)$$

The difference in the winding number between incident gravitational wave and absorbed gravitational wave is given by,

$$\Delta w=w_1-0=w_1. \quad ...(14)$$

In equation (14), we have to take the absolute value of Δw since w_1 can be positive or negative depending upon whether the field is moving counter-clockwise or clockwise along the closed string. And the coupling constant of the gravitational wave absorbed by the black hole is given below as:

$$g_2=l_p/w_2\, l_s. \quad ...(15)$$

In equation (15), when $w_2=0, g_2=\infty$. The change in Hamiltonian energy of the closed string in the case of absorption of gravitational waves by the black hole can be given using equation (6) as:

$$\delta H=(w_1\, R)/(l_s^2)+n_1\ (l_s^2)/R. \quad ...(16)$$

In equation (16), we take absolute value of w_1 since it may be positive or negative. And hence, using T- duality of string theory i.e. w~n in equation (16) the Hamiltonian energy difference for the absorbed gravitational waves by the black hole can be given as:

$$\delta H=1/g_1\ (R/(l_s^2)+(l_s^2)/R)\ l_p/l_s\,. \quad ...(17)$$

Using S-duality in equation (17), the Hamiltonian energy difference for the absorbed gravitational waves by the black hole in string theory having strong coupling constant $g_s=1/g_1$ is equivalent to string theory having weak coupling constant g_1. Thus, the Hamiltonian energy difference for the absorbed gravitational waves by the black hole using S-duality in string theory can be given by replacing $1/g_1$ with g_s in equation (17) as:

$$\delta H=g_s\ (R/(l_s^2)+(l_s^2)/R)\ l_s/l_p. \quad ...(18)$$

In equation (18), as $g_s \sim 1/g_1$ we will have to reverse l_p/l_s as l_s/l_p.

Coupling Constants of Gravitational Waves in String Theory in Terms of Winding Number of Closed String using S-Duality and T-Duality with Cyclic Vectors and Metric Spaces

In the scattering of gravitational waves by the black hole, we have winding numbers of the closed string $w_1,w_2\neq0$. So, the coupling constants of the closed string incident on the black hole is calculated as $g_1=l_p/w_1\, l_s$ and the value of the coupling constants of the closed string for gravitational waves scattered by the black holes calculated $g_2=l_p/w_2\, l_s$. We will take the values of winding numbers during the scattering of gravitational waves by the black hole$w_1,w_2=1,2,\ldots$ So, the values of the coupling constants of the closed strings will be g_1, $g_2=l_p/l_s$ [1/1,1/2,1/3,] which are weak coupling constants during the scattering of gravitational waves by the black hole gives us a sequence of cyclic vectors. Other values of w_1

and w_2 which we can take in equations (2) and (3) are w_1, $w_2=1^2,2^2$, we can also take the values w_1, $w_2=1^3,2^3$, and finally the values of w_1 and w_2 can be generalised as w_1, $w_2=1^p,2^p$, In equations (2) and (3), other values of the coupling constants of the closed string incident on the black hole can be given as $g_1,g_2=lp/l_s$ [$1/1^2$,$1/2^2$,$1/3^2$, we can also take the values $g_1,g_2=l_p/l_s$ [$1/1^3,1/2^3,1/3^3$, and finally the values of g_1 and g_2 can be generalised as g_1, $g_2=l_p/l_s$ [$1/1^p,1/2^p,1/3^p$. So, the weak coupling constants of gravitational waves as closed strings during the scattering of gravitational waves by the black hole can be generalised as metric spaces. Using S-duality, we can say that we will have equivalent string theory of strong coupling constants given as $g_{s1}=1/g_1$ and $g_{s2}=1/g_2$ So, we will have string theory having strong coupling constants equivalent to string theory having weak coupling constants existing during the scattering of gravitational waves by the black hole using S-duality. We can get the generalised values of the strong coupling constants g_{s1} & g_{s2} corresponding to weak coupling constants g_1 and g_2 as g_{s1}, $g_{s2}=l_s/l_p$ [$1^p,2^p,3^p$] which is the case of absorption of gravitational waves by the black hole. The various sequence of gravitational wave coupling constants obtained from the calculations predicts the various sequence of energies of the gravitational waves which depend upon the winding number of the closed string. Our results also provide us clues to the various gravitational coupling constants as explained in reference 64 predicting variations in gravitational coupling constants with the variations in interactions.

Conclusions

Coupling constants of gravitational waves in string theory in terms of winding number of closed string with cyclic vectors and metric spaces using S-duality and T-duality in string theory have been found in this paper. During the scattering of gravitational waves, we always have non-zero value of the winding numbers. During the scattering of gravitational waves, either the winding number is increased or decreased or remains same as that of the incident gravitational wave. The coupling constants of gravitational waves with black hole for all the three cases during the scattering of gravitational waves by the black hole considering gravitational wave as closed string has been calculated by us using equations (2) and (3). Equation (12) gives us the value of the coupling constant $g_2=g_1=g$ of the quantized form of gravitational wave as closed string which is scattered by the black hole with the same winding number initially it has. The values of the coupling constants of the closed strings can be generalised as metric spaces. During the absorption of gravitational wave by the black hole, the winding number of closed string becomes zero and the coupling constant of the gravitational wave absorbed by the black hole tends to infinity. Scattering of gravitational waves by the black can be studied in 26-dimensional bosonic string theory. But 26 dimensions is not enough to study the absorption. So, there is need of one extra dimension to study absorption of gravitational waves by the black hole in bosonic string theory. Using T- duality of string theory i.e. w~n in equation (6), the Hamiltonian energy difference for the scattered gravitational waves for the first two cases are given in equations (7) and (10) whereas $\delta H=0$ in third case during scattering of gravitational waves by the black hole. The change in Hamiltonian energy of the closed string in the case of absorption of gravitational waves by the black hole has been given in equation (16).Using T- duality of string theory i.e. w~n in equation (16) the Hamiltonian energy difference for the absorbed gravitational waves by the black hole as given by equation (17). We have got our results that using S-duality in equation (17), the Hamiltonian energy difference for the absorbed gravitational waves by the black hole in string theory having strong coupling constant $g_s=1/g_1$ is equivalent to string theory having weak coupling constant g_1 and hence the Hamiltonian energy difference for the absorbed gravitational waves by the black hole using S-duality in string theory can be given by replacing $1/g_1$ with gs in equation (17) which is given by equation (18). We can also study the scattering of gravitational waves by the black hole in 10-dimensional string theory. But we will need one extra dimension to explain the absorption of gravitational waves by the black hole which can be explained in 11-dimensional superstring theory or M-Theory. Finally, we have conclusion that the scattering of gravitational waves by black hole and absorption of gravitational waves by the black hole can be explained as two different aspects of the same string theory using S-duality that are equivalent to each other. We hope that these results will be verified soon.

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Conflict of Interest

The authors have no conflict of interest.

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