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On the Boundary Value Problem of a Finite Isotropic Wedge Under Anti Plane Deformation

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Abstract

A boundary value problem for a finite isotropic wedge under anti plane deformation has been solved. Depending on the boundary data on the radial edges, two cases are considered. The boundary data are: traction free-fixed displacement and traction-fixed displacement. The solution is accessed using the finite Mellin transform to obtain a full field solution for the stresses and displacements in the configuration through which the order of singularities due to the geometry are obtained for the cases considered.

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Introduction

Wedges are important geometry in most technical and engineering works, hence the need to understand their structural behavior.[14] William (1952) solved the problem of stress singularities at an apex of an isotropic elastic wedge using the method of Eigen function-expansion and obtained stresses at the wedge apex that is proportional to $r^{-\lambda s}$. Wedge apex and the stress singularities under various loading conditions was studied by [4]Demsey and Sinclair (1979). [6]Kargarnovin, Shahani and Fariborz (1997) analyzed the stress distribution in a wedge with finite radius subjected to anti plane shear deformation. They obtained the singularities at wedge apex under various loading as a dependent of apex angle. [8]Ma and Hour (1990) analyzed an anti plane shear deformation problem of composite anisotropic wedge with perfect bond along the interface by adopting Mellin transform in combination of stress functions. They assumed that the order of singularities is $O(r^{-\lambda s})$ as $r \to 0$. [10]Shahani (2007) extracted an expression for the singularities at the wedge apex by solving an anti plane shear deformation of a finite wedge using the finite Mellin transform. It was found that the necessary condition for singularities depends on the geometry. It was

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shown that stress fields are bounded for $0 < \alpha < \frac{\pi}{2}$, but for $\frac{\pi}{2} < \alpha < 2\alpha$ where α is the apex angle, the strength of geometry singularity is $\lambda_s = 1-\pi/2$. [2]Adimoha, Nnadi, Emenogu and Osu (2024) adopted Mellin transform in conjunction with the Wiener-Hopf technique to analyze the displacements everywhere in a finite isotropic cracked wedge under anti plane deformation. [3]Chih, Chein and Chien (2009) considered a finite a finite isotropic wedge under anti plane shear with a non zero anti plane shear load in the radial direction. The boundary data on the circular arc of the wedge were either fixed or traction-free. [1]Adimoha, Nnadi, Osu and Nwafor (2024) solved a mixed boundary value problem for a finite isotropic wedge under anti plane shear deformation using the Mellin transform, and obtained a closed form solution for the stresses and displacements. They articulated the stress intensity factor for the configuration which depends on the material constants.

Materials and Methods

In this paper, a parameterized elastic finite isotropic wedge configurations were considered. The geometrical configurations were subjected to an anti plane shear deformation for the two different cases established by different boundary conditions. The schematic cross section of the wedges were shown in figures 1 and 2, respectively.

The parameterization of the geometry were performed on their main parameters: stresses, displacements, apex angles, traction free-fixed displacement and traction-fixed displacement.

The materials were considered in an elastic formulation with their material properties μ and the apex angle α.

Two cases were considered, traction free-fixed displacement and traction-fixed displacement, respectively.

The methodology of the paper is owing to the fact that functions in elastic solids satisfies the equilibrium equation, as well as boundary conditions. The task was implemented as a boundary value problem with the mathematical problem supplemented by boundary conditions.

As a result of the nature of the boundary conditions, we adopted the finite Mellin transform of the second kind in conjunction with its inversion formula to obtain the convergence series form of the required fields, through which the fracture responses and the strength of singularities of the wedges were established.

Basic Equations for Cases Considered

A finite isotropic wedge of radius a and apex angle α under anti plane shear deformation is considered. Therefore, the only non-vanishing displacement component is the one in z-direction given by W(r,θ), which is a function of the in-plane coordinates r and θ. The non-vanishing components of the stresses are given by

$$\sigma_{\theta z}(r,\theta) = \frac{\mu}{r}\frac{\partial W(r,\theta)}{\partial \theta} \qquad ...(1)$$

$$\sigma_{rz}(r,\theta) = \mu\frac{\partial W(r,\theta)}{\partial r} \qquad ...(2)$$

where μ is the shear material constant. In the absence of body force, the equilibrium equation yields the Laplace equation in terms of the displacement by

$$\frac{\partial W(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial W(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 W(r,\theta)}{\partial \theta^2} = 0, 0 \le r \le a, 0 \le \theta \le 2\pi \qquad ...(3)$$

The boundary conditions are:

$$\sigma_{rz}(r,\theta)=0 \ ,0\le\theta\le\alpha \ ,0\le\alpha\le 2\pi \qquad ...(4)$$

$$W(a,\theta)=0 \ ,0\le\theta\le 2\pi \ ,0\le\alpha\le 2\pi \qquad ...(5)$$

The finite Mellin transform of the first and second kinds are to be employed in this analysis. The finite Mellin transform of the first kind is defined by

$$\hat{W}_1(s,\theta) = \int_0^a \left(\frac{a^{2s}}{r^{s+1}} - r^{s-1}\right) W(r,\theta)dr \qquad ...(6)$$

The finite Mellin transform of the second kind is defined by

$$\hat{W}_2(s,\theta) = \int_0^a \left(\frac{a^{2s}}{r^{s+1}} + r^{s-1}\right) W(r,\theta)dr \qquad ...(7)$$

If $\hat{W}_1$ (s,θ) and $\hat{W}_2$ (s,θ) are found, then W(r,θ) is obtained by the use of the Mellin inversion formula defined by

$$W(r,\theta) = \frac{(-1)^j}{2\pi i}\int_{c-i\infty}^{c+i\infty}\widehat{W}_j(s,\theta)r^{-s}ds\ , j = 1,2 \qquad \text{...(8)}$$

The application of (6) to the Laplace equation (3) and the use made of integration by parts and the Leibnitz rule for differentiating the integral yields

$$\frac{d^2\widehat{W}_1(s,\theta)}{d\theta^2} + s^2\widehat{W}_1(s,\theta) + 2a^s sW(a,\theta) = 0 \qquad \text{...(9)}$$

provided that

$$\lim_{r\to 0}\left[(a^{2s}r^{-s} - r^s)r\frac{\partial W(r,\theta)}{\partial r} + s(a^{2s}r^s + r^s)W(r,\theta)\right] = 0 \qquad \text{...(10)}$$

also,

application of (7) to (3) and use made of integration by parts and the Leibnitz rule for differentiating the integral produces

$$\frac{d^2\widehat{W}_2(s,\theta)}{d\theta^2} + s^2\widehat{W}_2(s,\theta) + 2a^{s+1}\frac{\partial W(a,\theta)}{\partial r} = 0 \qquad \text{...(11)}$$

provided that

$$\lim_{r\to 0}\left[(a^{2s}r^{-s} + r^s)r\frac{\partial W(a,\theta)}{\partial r} + s(a^{2s}r^{-s} - r^s)W(a,\theta)\right] = 0 \qquad \text{...(12)}$$

The range of values of c in the inversion formula (8), is the strip of regularity of W(s,θ). It is derived from the conditions (10) and (12).

The stress and displacement has the following asymptotic behaviors

$$\sigma_{rz}(r,\theta) = \sigma_{\theta z} = O(r^{\lambda s}, 0<\lambda<1, \text{ as } r\to 0 \qquad \text{...(13)}$$

$$W(r,\theta) = O(r^{1-\lambda s}), 0<\lambda<1, \text{ as } r\to 0 \qquad \text{...(14)}$$

Referring to (12) in view of (13) and (14) leads to

$$\lim_{r\to 0}\left[(a^{2s}r^{-s} + r^s)r^{1-\lambda} + s(a^{2s}r^{-s} - r^s)r^{1-\lambda}\right] =$$

$$\lim_{r\to 0}\left[(a^{2s}r^{-2s} + 1)r^{s+1-\lambda} + s(a^{2s}r^{-2s} - 1)r^{s+1-\lambda}\right] =$$

$$\lim_{r\to 0}[(1+s)a^{2s}r^{-2s} + 1 - s]r^{s+1-\lambda}$$

But $r < a$ implies $\frac{a}{r} > 1$ or $\left(\frac{a}{r}\right)^{2s} > 1$

Hence

$(1+s)a^{2s}r^{-2s} + (1-s) > (1+s) + (1-s) = 2$

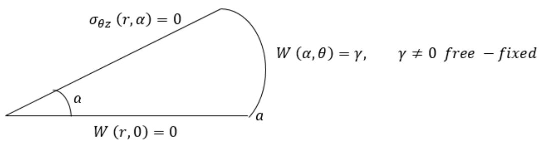
implies that

$r^{s+1-\lambda} \to 0$, as $r \to 0$

if

$$\text{Re}\,s + 1 - \lambda > 0 \qquad \text{...(15)}$$

or Res>λ-1, where Re means the real part of. Equations (10), (13) and (14) lead to the same result if Res≠0.



Problem 1
(Traction Free-Fixed Displacement)

By the use of (6) and (9) we get

$$\frac{d^2\widehat{W}(s,\theta)}{d\theta^2} + s^2\widehat{W}(s,\theta) = -2sa^s\gamma \qquad \text{...(16)}$$

$$\widehat{W}(s,0) = 0 \qquad \text{...(17)}$$

$$\frac{d\widehat{W}(s,\alpha)}{d\theta} = 0 \qquad \text{...(18)}$$

Consider the solution of (16) to be of the form

$$\widehat{W}(s,\theta) = A(s)cos\theta s + B(s)sin\theta s - \frac{2a^s\gamma}{s} \qquad \text{...(19)}$$

$$\frac{d\widehat{W}(s,\theta)}{d\theta} = -sA(s)sin\theta s + sB(s)cos\theta s$$

$$\widehat{W}(s,0) = A(S) - \frac{2a^s\gamma}{s} = 0$$

$$A(s) = \frac{2a^s\gamma}{s} \qquad \text{...(20)}$$

$$\frac{d\widehat{W}(s,\alpha)}{d\theta} = -sA(s)sin\alpha s + sB(s)cos\alpha s = 0$$

$$B(s) = \frac{A(s)sin\alpha s}{cos\alpha s} \qquad \text{...(21)}$$

Hence, substituting (20) and (21) into (19) yields

$$\widehat{W}(s,\theta) = \frac{2a^s\gamma}{s}cos\theta s + \frac{2a^s\gamma}{s}\frac{sin\alpha s sin\theta s}{cos\alpha s} - \frac{2a^s\gamma}{s}$$

$$= \frac{2a^s\gamma}{scos\alpha s}[cos\alpha s cos\theta s + sin\alpha s sin\theta s] - \frac{2a^s\gamma}{s}$$

$$\widehat{W}(s,\theta) = \frac{2a^s\gamma}{scos\alpha s}cos(\alpha-\theta)s - \frac{2a^s\gamma}{s} \qquad \text{...(22)}$$

Substituting (22) into (8) yields

$$W(r,\theta) = -\frac{2\gamma}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{cos(\alpha-\theta)s}{scos\alpha s}\left(\frac{r}{a}\right)^{-s}ds - \frac{2\gamma}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{1}{s}\left(\frac{r}{a}\right)^{-s}ds \qquad \text{...(23)}$$

The Bromwich integrands in (23) can be evaluated by Cauchy's residue theorem in accordance with Jordan's lemma. The second integrand has a pole at s=0 which leads to constants as r gets to the neighborhood of zero. These constants are ignored because the problem being solved is a Neumann boundary value problem for which constants do not affect solutions. Therefore, we consider the first integrand in (23) with simple poles at $s_n=\pm(2n-1)\,\pi/2\alpha$, $n=1,2,3\ldots$ Because of convergence as $r\to 0$, we give attention to $s_n=-(2n-1)\,\pi/2\alpha$, $n=1,2,3\ldots$

Therefore, residue theorem gives

$$W(r,\theta)=-\frac{4\alpha\gamma}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\alpha sin(2n-1)\frac{\pi}{2}sin(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{a}\right)^{(2n-1)\frac{\pi}{2\alpha}},r<a \quad \ldots(24)$$

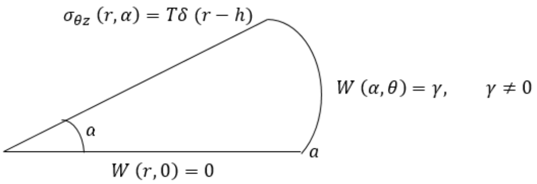
Utilizing (1), (2) and (24), the corresponding stress components reads

$$\sigma_{\theta z}(r,\theta)=-2\mu\gamma\sum_{n=1}^{\infty}(-1)^n\,\alpha sin(2n-1)\frac{\pi}{2}cos(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{a}\right)^{(2n-1)\frac{\pi}{2\alpha}-1},r<a \quad \ldots(25)$$

$$\sigma_{rz}(r,\theta)=-2\mu\gamma\sum_{n=1}^{\infty}(-1)^n\,\alpha sin(2n-1)\frac{\pi}{2}sin(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{a}\right)^{(2n-1)\frac{\pi}{2\alpha}-1},r<a \quad \ldots(26)$$

Having obtained (25) and (26), if s_1 is the least of the poles with $|s_1|<1$ and $\lambda_{s1}=1-s_1$, then λ_{s1} is the order or strength of the stress singularity at the apex of the wedge. Whether or not a stress singularity occurs at the wedge apex depends on the fact that the inequality $|s_1|<1$ holds. Therefore, $0\le\alpha<2\pi$ implies $0<\alpha<\pi/2$ and $\pi/2<\alpha<2\pi$. Also, $0<\alpha<\pi/2$ implies $\pi/2\alpha>1$. That is $(2n-1)\,\pi/2\alpha>2n-1$ or $(2n-1)\,\pi/2\alpha-1>2(2n-1)$. Thus, for n=1, $\pi/2\alpha-1>0$ or $|s_1|>1$. For this case, the fields are bounded.

But, if $\pi/2<\alpha$, then $\pi/2\alpha<1$ implies $(2n-1)\,\pi/2\alpha<2n-1$ and $(2n-1)\,\pi/2\alpha-1<2(n-1)$. Thus, for n=1, we have $s_1=\pi/2\alpha<1$. Therefore, singularity occurs for all $\alpha>\pi/2$.



Problem 2
(Traction-Fixed Displacement)

from problem (1)

$$\frac{d^2\widehat{W}(s,\theta)}{d\theta^2}+s^2\widehat{W}(s,\theta)+2a^s s\gamma=0 \quad \ldots(27)$$

$$\hat{W}(s,0)=0 \quad \ldots(28)$$

$$\frac{r}{\mu}\sigma_{\theta z}(r,\alpha)=\frac{T}{\mu}\delta(r-h) \quad \ldots(29)$$

$$\frac{\partial\widehat{W}(s,0)}{\partial\theta}=g(s)\frac{T}{\mu}h^s$$

$$g(s)=\left[\left(\frac{h}{a}\right)^{2s}-1\right]\frac{1}{h} \quad \ldots(30)$$

Let the solution of (27) be

$$\hat{W}(s,\theta)=A(s)\cos\theta s+B(s)\sin\theta s-2a^s\gamma)/s \quad \ldots(31)$$

then

$$\frac{d\widehat{W}(s,\theta)}{d\theta}=-sA(s)sin\theta s+sB(s)cos\theta s$$

$$\hat{W}(s,\theta)=A(s)-(2a^2\gamma)/s=0$$

$$A(s)=(2a^2\gamma)/s \quad \ldots(32)$$

$$(d\hat{W}(s,\alpha)/d\theta=-sA(s)\sin\alpha s+sB(s)\cos\alpha s=g(s)$$

then

$$B(s)=(g(s)/s\cos\alpha s+A(s)\sin\alpha s/\cos\alpha s \quad \ldots(33)$$

Therefore, using (31), (32) and (33) yields

$$W(r,\theta)=\frac{2\gamma}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left[\frac{\cos(\alpha-\theta)s}{scos\alpha s}-\frac{1}{s}\right]\left(\frac{r}{a}\right)^{-s}ds+\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{T}{\mu}\frac{g(s)sin\theta s}{scos\alpha s}\left(\frac{r}{h}\right)^{-s}ds \quad \ldots(34)$$

Adopting the inverse Mellin transform defined by (8), we obtain

$$W(r,\theta)=\frac{2\gamma}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left[\frac{\cos(\alpha-\theta)s}{scos\alpha s}-\frac{1}{s}\right]\left(\frac{r}{a}\right)^{-s}ds+\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{T}{\mu}\frac{g(s)sin\theta s}{scos\alpha s}\left(\frac{r}{h}\right)^{-s}ds \quad \ldots(35)$$

The Bromwich integral in (35) can be evaluated by the residue method. The simple poles at s=0 are dropped because they produce constants which do not alter the solution of a Neumann boundary value problem. For the sake of convergence, the residue at simple poles given by $s_n=-(2n-1)\,\pi/2\alpha$, $n=1,2,3\ldots$ to obtain the required fields. Therefore, sum of residue gives for the first and second integrands respectively yields

$$W(r,\theta)=-\frac{4\gamma}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\sin(2n-1)\frac{\pi}{2\alpha}\sin(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{a}\right)^{(2n-1)\frac{\pi}{2\alpha}},r<a \quad \ldots(36)$$

$$W(r,\theta)=-\frac{2T}{\mu\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\left\{g(s)\sin\,(2n-1)\frac{\pi\theta}{2\alpha}\right\}\left(\frac{r}{h}\right)^{(2n-1)\frac{\pi}{2\alpha}},r<h \quad \ldots(37)$$

Then, utilizing (36) and (37), their corresponding stresses are obtained as

$$\sigma_{\theta z}(r,\theta) = -\frac{2\gamma\mu}{\alpha}\sum_{n=1}^{\infty}(-1)^n \sin(2n-1)\frac{\pi}{2\alpha}\cos(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{a}\right)^{(2n-1)\frac{\pi}{2\alpha}-1}, r < a \quad ...(38)$$

$$\sigma_{rz}(r,\theta) = -\frac{2\gamma\mu}{\alpha}\sum_{n=1}^{\infty}(-1)^n \sin(2n-1)\frac{\pi}{2\alpha}\sin(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{a}\right)^{(2n-1)\frac{\pi}{2\alpha}-1}, r < a \quad ...(39)$$

also

$$\sigma_{\theta z}(r,\theta) = -\frac{T}{\alpha}\sum_{n=1}^{\infty}(-1)^n g(s)\cos(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{h}\right)^{(2n-1)\frac{\pi}{2\alpha}-1}, r < h \quad ...(40)$$

$$\sigma_{rz}(r,\theta) = -\frac{T}{\alpha}\sum_{n=1}^{\infty}(-1)^n g(s)\sin(2n-1)\frac{\pi\theta}{2\alpha}\left(\frac{r}{h}\right)^{(2n-1)\frac{\pi}{2\alpha}-1}, r < h \quad ...(41)$$

Results

Considering (38),(39),(40) and (41), if (2n-1)π/2α>1, then (2n-1) π/2α-1>0, so that the power of (r/a) or (r/h) in $\sigma_{\theta z}$ (r,θ) or σ_{rz} (r,θ) for the fields from both integrals will be positive and therefore bounded. This happens when 0<α<π/2. as r→0. If on the other hand, π/2<α<2π, then π/2α<1<2π/α which implies that π/2α<1. Then, s_1<1 will hold, so there exist stress singularity at the wedge apex. Therefore, the stress fields are unbounded as r→0.

The asymptotic behaviour (13) yields the strength of geometric singularity as

$$\lambda_{s1} = 1 - \pi/2\alpha \quad ..(42)$$

Discusion

From the analysis and (42), it can be seen that when α approaches critical values of 2π/3,3π/4 and π ,which is a case of circular shaft with a radial crack, then strength of singularity exhibits significant variations, specifically decreasing to 3/4, 2/3 and 1/2 respectively. This shows the relationship between the wedge geometry and the resultant stress distribution around singularities. Also, the implication of this research extends to the importance of understanding singularities in the design of structures in practical engineering application.

Conclusion

A boundary value problem for a finite isotropic wedge under anti plane shear deformation has been solved in this paper. The finite Mellin transform of the second kind was employed to solve a two-dimensional Neumann boundary value problem in terms of the only non-zero displacement component. Two different cases of traction free-fixed displacement and traction –fixed displacement were examined. Exact solutions for the required fields were obtained for both cases and the effects of geometry on the strength of geometric singularity were also examined.

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This statement does not apply to this article.

Ethics Statement

This research did not involve human participants, animal subjects, or any material that requires ethical approval.

Informed Consent Statement

This study did not involve human participants, and therefore, informed consent was not required

Author Contributions

- **Bright Okore Osu And Bright Adinchezo Adimoha:** Conceptualization, Formulation, Analysis and Supervision
- **Bright Adinchezo Adimoha, Bright Adinchezo Adimoha, Bright Adinchezo Adimoha and Murphy Emeke Egwe:** Validation and Visualization

- **Murphy Emeke Egwe and Murphy Emeke Egwe:** Writing the original draft, Review and Editing
- **Bright Adinchezo Adimoha, Bright Adinchezo Adimoha, George Ndubueze Emenogu and George Ndubueze Emenogu:** Funding

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