On The Efficiency of Ratio Estimators of Finite Population Mean using Auxiliary Information

JAMIU OLASUNKANMI MUILI*, AHMED AUDU2 and IBRAHIM YUNUSA ADAMU3

1Department of Mathematics, Kebbi State University of Science and Technology Aliero, Nigeria.
2Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria.
3Department of Mathematics and Statistics, Federal Polytechnic Nasarawa, Nasarawa State, Nigeria.

Abstract
Ratio estimation is technique that usages available auxiliary information which is certainly correlated with study variable. In this study, class of ratio-type estimators of finite population mean has been anticipated to solve delinquent of estimation of population mean. Properties of anticipated estimators namely Bias & Mean Square Error were acquired up to first order of approximation & condition for their efficiency over some existing estimators was also established. The results show that anticipated estimators are enhanced & proficient (minimum mean square errors) than other estimators with the highest precision.

Introduction
Usage of auxiliary information is made through the ratio & product techniques of estimation to enhance estimates of population mean. Estimation of population mean of variable of interest with higher precision is unremitting issue in sample survey. So, precision could be increased by the used of apposite estimation procedure which consumes auxiliary information which is meticulously associated to variable of interest. In ratio method of estimation, auxiliary information is available which is linearly related to the variable of study. The population parameters such as populations' median, coefficient of kurtosis, skewness, coefficient of variation, decile, quartile, correlation, etc are auxiliary variables. Efficiency of estimators of population parameters can be increased by suitable usage of auxiliary information in relationship with auxiliary variable. Cochran1 came up with what is known as ratio-type estimator for estimation of population mean which is more competent than sample mean. Many authors have used different auxiliary information inorder to
enhance the precision of the estimates by using prior knowledge of population parameters. Researchers in sample survey like Kadilar and Cingi developed classes of ratio estimators using known auxiliary information on coefficients of variation, & kurtosis. Abid et al. also suggested set of ratio-type estimators for the population mean using non-conventional location parameters like mid-range, and tri-mean as auxiliary information. Other researchers are Upadhyaya and Singh, Subramani and Kumarapadiyan, Subramani and Kumarapadiyan, Subramani and Kumarapadiyan, Subramani and Kumarapadiyan, Jeelani et al., and Nasir et al.

The objective of this study is to develop innovative set of ratio-type estimators to increase precision of estimates of population mean using known auxiliary information.

Let \( U = (U_1, U_2, ..., U_N) \) be finite population having \( N \) units & each \( U_i = (X_i, Y_i), i = 1, 2, 3, ..., N \) has pair of values. \( Y \) is study variable & \( X \) is auxiliary variable which is associated (correlated) with \( Y \), in which \( x = (x_1, x_2, ..., x_n) \) & \( y = (y_1, y_2, ..., y_n) \) are the \( n \) sample values. & \( \bar{y} \) is sample mean of variable of interest & \( \bar{x} \) is sample mean of auxiliary variable. \( s_{1}^{2} \) is sample mean square of study variable & \( s_{2}^{2} \) is sample mean square of auxiliary variable based on random sample of size \( n \) drawn without replacement. and \( s_{1}^{2} \) is population mean square of study variable and \( s_{2}^{2} \) is population mean square of auxiliary variable. Following are other symbols used in this study.

\[
\begin{align*}
Y & : \text{Study variable} \\
N & : \text{Population size} \\
X & : \text{Auxiliary variable} \\
n & : \text{Sample size} \\
\bar{y} & : \text{Sample means of study variable} \\
\bar{x} & : \text{Sample means of auxiliary variable} \\
\hat{Y} & : \text{Population means of study variable} \\
\hat{X} & : \text{Population means of auxiliary variable} \\
S_{pw} & : \text{Probability weighted moments} \\
p & : \text{Coefficient of correlation} \\
C_r & : \text{Coefficient of variation of study variable} \\
Q_3 & : \text{The upper quartile} \\
C_v & : \text{Coefficient of variation of auxiliary variable} \\
QD & : \text{Population Quartile Deviation} \\
\beta_1 & : \text{Coefficient of skewness} \\
\beta_2 & : \text{Coefficient of kurtosis} \\
G & : \text{Gini’s Mean Difference} \\
TM & : \text{Tri-Mean} \\
M_d & : \text{Median} \\
MR & : \text{Population mid-range} \\
HR & : \text{Hodges-Lehman estimator} \\
D & : \text{Downton’s Method} \\
\end{align*}
\]

The Existing Estimators in Literature

Cochran developed the conventional ratio estimator for estimating population mean \( (\hat{Y}) \) of study variable \( (Y) \) given as:

\[
\hat{y} = \hat{Y} - \frac{X}{\bar{Y}} \bar{X} X
\]

where \( \bar{R} = \frac{\bar{y}}{\bar{X}} \)

\[
Bias(\hat{Y}) = \gamma \left( RS_{1}^{2} - \rho S_{2} S_{3} \right) \quad \ldots (1.2)
\]

\[
MSE(\hat{Y}) = \gamma \left( S_{1}^{2} + R^2 S_{2}^2 - 2 R \rho S_{2} S_{3} \right) \quad \ldots (1.3)
\]

Nasir et al modified class of ratio type estimators for finite population mean consuming known values of coefficient of variation \( (C_v) \) & decile mean of auxiliary information, biases, constants and mean square errors are given as:

\[
\hat{Y}_1 = \frac{\hat{y} + N(\hat{X} - \bar{X})}{(X + DM)} \quad \ldots (1.4)
\]

\[
\hat{Y}_2 = \frac{\bar{Y} + b(\hat{X} - \bar{X})}{(X_{\bar{C}} + DM)} \quad \ldots (1.5)
\]

\[
\hat{Y}_3 = \frac{\bar{Y} + b(\hat{X} - \bar{X})}{(X_{\bar{C}} + DM)} \quad \ldots (1.6)
\]

\[
Bias(\hat{Y}_i) = -\gamma \left[ \frac{S_{3}}{\hat{Y}_i} \right] R_i, \quad \text{where } i = 1, 2, 3 \quad \ldots (1.7)
\]

\[
MSE(\hat{Y}_i) = -\gamma \left[ R_i S_{1}^{2} + S_{3}^{2} (1 - \rho^2) \right] where i = 1, 2, 3 \quad \ldots (1.8)
\]

Subzar developed a class ratio type estimators
using linear combination of different known population parameters given as:

\[ \hat{\mu}_1 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.9)

\[ \hat{\mu}_2 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.11)

\[ \hat{\mu}_3 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.12)

\[ \hat{\mu}_4 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.13)

\[ \hat{\mu}_5 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.14)

\[ \hat{\mu}_6 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.15)

\[ \hat{\mu}_7 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.16)

\[ \hat{\mu}_8 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.17)

\[ \hat{\mu}_9 = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.18)

\[ \hat{\mu}_{10} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.19)

\[ \hat{\mu}_{11} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.20)

\[ \hat{\mu}_{12} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.21)

\[ \hat{\mu}_{13} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.22)

\[ \hat{\mu}_{14} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.23)

\[ \hat{\mu}_{15} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.24)

\[ \hat{\mu}_{16} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.25)

\[ \hat{\mu}_{17} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.26)

\[ \hat{\mu}_{18} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.27)

\[ \hat{\mu}_{19} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (1.28)

Proposed Estimator

Motivated by the work of Subzar et al.\textsuperscript{13}, we proposed ratio-type estimators for estimating population mean using value of hogdes-lehmann as:

\[ \hat{\mu}_{12} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.1)

\[ \hat{\mu}_{13} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.2)

\[ \hat{\mu}_{14} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.3)

\[ \hat{\mu}_{15} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.4)

\[ \hat{\mu}_{16} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.5)

\[ \hat{\mu}_{17} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.6)

\[ \hat{\mu}_{18} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.7)

\[ \hat{\mu}_{19} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.8)

\[ \hat{\mu}_{20} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.8)

\[ \hat{\mu}_{21} = \frac{\bar{x} + h(\bar{x} - \bar{x})}{(\bar{x} + \bar{v}_1)} \] ... (2.8)

where

\begin{align*}
\eta_1 &= |\bar{x} + G|, \quad \eta_2 = |\bar{x} + D|, \quad \eta_3 = |\bar{x} + D|, \quad \eta_4 = |\bar{x} + G|, \quad \eta_5 = |\bar{x} + D|, \quad \eta_6 = |\bar{x} + G|, \quad \eta_7 = |\bar{x} + D|, \quad \eta_8 = |\bar{x} + G|, \quad \eta_9 = |\bar{x} + D|, \quad \eta_{10} = |\bar{x} + G|, \quad \eta_{11} = |\bar{x} + D|, \quad \eta_{12} = |\bar{x} + G|, \quad \eta_{13} = |\bar{x} + D|, \quad \eta_{14} = |\bar{x} + G|, \quad \eta_{15} = |\bar{x} + D|, \quad \eta_{16} = |\bar{x} + G|, \quad \eta_{17} = |\bar{x} + D|, \quad \eta_{18} = |\bar{x} + G|, \quad \eta_{19} = |\bar{x} + D|, \quad \eta_{20} = |\bar{x} + G|, \quad \eta_{21} = |\bar{x} + D|.
\end{align*}

**Bias** of \( \hat{\mu}_j \) where \( j = 4, 5, \ldots, 21 \)

\[ \text{Bias}(\hat{\mu}_j) = \frac{\eta_j S^2_j}{\bar{x} + \bar{v}_1} \] ... (2.29)

**MSE** of \( \hat{\mu}_j \) where \( j = 4, 5, \ldots, 21 \)

\[ \text{MSE}(\hat{\mu}_j) = \frac{\eta_j S^2_j + S^2_j (1 - \rho^2)}{\bar{x} + \bar{v}_1} \] ... (2.31)
In order to derive bias & mean square error, \( \kappa = \frac{\hat{y} - \bar{y}}{\bar{y}} \) and \( \mu = \frac{\hat{x} - \bar{x}}{\bar{x}} \) such that

\[
\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_{i} \quad \text{and} \quad \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_{i},
\]
from the definition of \( \hat{\epsilon}_{0} \) and \( \hat{\epsilon}_{1} \), we obtain

\[
E(\hat{\epsilon}_{0}) - E(\hat{\epsilon}_{1}) - 0, \quad E(\hat{\epsilon}_{1}) - \gamma C_{j}^{2} \quad \text{and} \quad E(\hat{\epsilon}_{0}) - \gamma C_{j}^{2} - \mu C_{j}^{2} C_{s}^{1}
\]
\[
\text{(2.23)}
\]

\[
\text{Bias}(\hat{\epsilon}_{i}) = \frac{\hat{\epsilon}_{i} - \epsilon_{i}}{\epsilon_{i}}, \quad i = (1, 2, 3, 4, 5, 6, \ldots)
\]
\[
\text{(2.24)}
\]

\[
\text{MSE}(\hat{\epsilon}_{i}) = \frac{\hat{\epsilon}_{i}^{2}}{\hat{\epsilon}_{i}^{2}}, \quad i = (1, 2, 3, 4, 5, \ldots)
\]
\[
\text{(2.25)}
\]

where

\[
\hat{\epsilon}_{i} = \frac{\hat{y}}{\bar{y}} - \frac{y}{\bar{y}}, \quad \hat{\epsilon}_{i} = \frac{\hat{x}}{\bar{x}} - \frac{x}{\bar{x}}, \quad \hat{\epsilon}_{i} = \frac{\hat{y}}{\bar{y}} - \frac{y}{\bar{y}}, \quad \hat{\epsilon}_{i} = \frac{\hat{x}}{\bar{x}} - \frac{x}{\bar{x}}, \quad \hat{\epsilon}_{i} = \frac{\hat{y}}{\bar{y}} - \frac{y}{\bar{y}}, \quad \hat{\epsilon}_{i} = \frac{\hat{x}}{\bar{x}} - \frac{x}{\bar{x}}.
\]

Percentage Relative Efficiency (PRE) is given as

\[
\text{PRE} = \left( \frac{\text{MSE}(\hat{\epsilon}_{i})}{\text{MSE}(\hat{\epsilon}_{j})} \right) \times 100
\]
\[
\text{(2.26)}
\]

\( \hat{\epsilon}_{j} \) are estimators in this study.

**Efficiency Comparisons**

Efficiencies of suggested estimators are compared with efficiencies of existing estimators in study

\[
\text{MSE}(\hat{\epsilon}_{i}) < \text{MSE}(\hat{\epsilon}_{j}) \quad \text{if}, \quad i = 1, 2, \ldots, 18
\]
\[
\left( R_{1}^{2} S_{1}^{2} + S_{1}^{2}(1 - \rho^{2}) \right) < \left( S_{2}^{2} + R_{1}^{2} S_{2}^{2} - 2R_{1}S_{1}S_{2} \right)
\]
\[
\text{(2.27)}
\]

The \( \hat{\epsilon}_{i} \) of proposed estimators of population mean is more efficient than \( \hat{\epsilon}_{j} \) if,

\[
\text{MSE}(\hat{\epsilon}_{i}) < \text{MSE}(\hat{\epsilon}_{j}) \quad \text{if}, \quad i = 1, 2, \ldots, 18 
\]
\[
\left( R_{1}^{2} S_{1}^{2} + S_{1}^{2}(1 - \rho^{2}) \right) < \left( R_{1}^{2} S_{2}^{2} + S_{2}^{2}(1 - \rho^{2}) \right)
\]
\[
\text{(2.28)}
\]

When conditions (2.27), and (2.28) are contented, conclusion will be made that anticipated estimators are better and relatively efficient than other estimators in the study.

**Empirical Study**

To evaluate performance of anticipated estimators, following real populations are used.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Population I</th>
<th>Population II</th>
<th>Population III</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>34</td>
<td>34</td>
<td>80</td>
</tr>
<tr>
<td>n</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>( \bar{Y} )</td>
<td>856.4117</td>
<td>856.4117</td>
<td>5182.63</td>
</tr>
<tr>
<td>( \bar{X} )</td>
<td>199.4412</td>
<td>208.8823</td>
<td>1126.463</td>
</tr>
<tr>
<td>P</td>
<td>0.4453</td>
<td>0.4491</td>
<td>0.941</td>
</tr>
<tr>
<td>S_{1}</td>
<td>733.1407</td>
<td>733.1407</td>
<td>1835.659</td>
</tr>
<tr>
<td>C_{1}</td>
<td>0.8561</td>
<td>0.8561</td>
<td>0.354193</td>
</tr>
<tr>
<td>S_{2}</td>
<td>150.2150</td>
<td>150.5059</td>
<td>845.610</td>
</tr>
<tr>
<td>C_{2}</td>
<td>0.7531</td>
<td>0.7205</td>
<td>0.7506772</td>
</tr>
<tr>
<td>( \beta_{2} )</td>
<td>1.0445</td>
<td>0.0978</td>
<td>-0.063386</td>
</tr>
<tr>
<td>( \beta_{1} )</td>
<td>1.1823</td>
<td>0.9782</td>
<td>1.050002</td>
</tr>
<tr>
<td>M_{1}</td>
<td>142.5</td>
<td>150</td>
<td>757.5</td>
</tr>
<tr>
<td>MR</td>
<td>320</td>
<td>284.5</td>
<td>1795.5</td>
</tr>
<tr>
<td>QD</td>
<td>184</td>
<td>80.25</td>
<td>588.125</td>
</tr>
<tr>
<td>G</td>
<td>162.996</td>
<td>155.446</td>
<td>901.081</td>
</tr>
<tr>
<td>D</td>
<td>144.481</td>
<td>140.891</td>
<td>801.381</td>
</tr>
<tr>
<td>S_{rw}</td>
<td>206.944</td>
<td>199.961</td>
<td>791.364</td>
</tr>
<tr>
<td>DM</td>
<td>206.944</td>
<td>234.82</td>
<td>1150.7</td>
</tr>
</tbody>
</table>
Table 2: Constant and Bias of Some Selected Existing and Proposed Estimators

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Constant</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pop-I</td>
<td>Pop-II</td>
</tr>
<tr>
<td></td>
<td>4.294</td>
<td>1.9301</td>
</tr>
<tr>
<td></td>
<td>1.806</td>
<td>1.6051</td>
</tr>
<tr>
<td></td>
<td>1.289</td>
<td>1.1703</td>
</tr>
<tr>
<td></td>
<td>1.6960</td>
<td>1.6651</td>
</tr>
<tr>
<td></td>
<td>1.7606</td>
<td>1.7136</td>
</tr>
<tr>
<td></td>
<td>1.5602</td>
<td>1.5324</td>
</tr>
<tr>
<td></td>
<td>1.8955</td>
<td>1.9263</td>
</tr>
<tr>
<td></td>
<td>1.9764</td>
<td>1.9915</td>
</tr>
<tr>
<td></td>
<td>1.7274</td>
<td>1.751</td>
</tr>
<tr>
<td></td>
<td>0.9671</td>
<td>0.9633</td>
</tr>
<tr>
<td></td>
<td>1.0148</td>
<td>0.9997</td>
</tr>
<tr>
<td></td>
<td>0.8701</td>
<td>0.8666</td>
</tr>
<tr>
<td></td>
<td>1.1177</td>
<td>1.1672</td>
</tr>
<tr>
<td></td>
<td>1.1818</td>
<td>1.2211</td>
</tr>
<tr>
<td></td>
<td>0.9902</td>
<td>1.0283</td>
</tr>
<tr>
<td></td>
<td>1.4153</td>
<td>1.5353</td>
</tr>
<tr>
<td></td>
<td>1.4752</td>
<td>1.3979</td>
</tr>
<tr>
<td></td>
<td>1.2908</td>
<td>1.2329</td>
</tr>
<tr>
<td></td>
<td>1.6021</td>
<td>1.5977</td>
</tr>
<tr>
<td></td>
<td>1.6793</td>
<td>1.6603</td>
</tr>
<tr>
<td></td>
<td>1.4444</td>
<td>1.4326</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0335</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0369</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.0261</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0334</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0368</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.0261</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0336</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0371</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.0262</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0336</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0370</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.0262</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0336</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0370</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.0262</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0336</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0370</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.0262</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0336</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0370</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.0262</td>
</tr>
</tbody>
</table>

Table 2 shows the constant and bias of estimators
Table 3: MSE and PRE of Estimators

<table>
<thead>
<tr>
<th>Estimator</th>
<th>MSE</th>
<th>PRE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pop-I</td>
<td>Pop-II</td>
</tr>
<tr>
<td>( \hat{y}_1 )</td>
<td>10960.76</td>
<td>10539.27</td>
</tr>
<tr>
<td>( \hat{y}_2 )</td>
<td>10934.74</td>
<td>10571.58</td>
</tr>
<tr>
<td>( \hat{y}_3 )</td>
<td>10386.83</td>
<td>10030.11</td>
</tr>
<tr>
<td>( \hat{y}_4 )</td>
<td>9644.04</td>
<td>9472.95</td>
</tr>
<tr>
<td>( \hat{y}_5 )</td>
<td>10208.16</td>
<td>10333.07</td>
</tr>
<tr>
<td>( \hat{y}_6 )</td>
<td>10311.83</td>
<td>9929.39</td>
</tr>
<tr>
<td>( \hat{y}_7 )</td>
<td>10540.91</td>
<td>10656.74</td>
</tr>
<tr>
<td>( \hat{y}_8 )</td>
<td>10686.6</td>
<td>10683.87</td>
</tr>
<tr>
<td>( \hat{y}_9 )</td>
<td>10258.09</td>
<td>10264.06</td>
</tr>
</tbody>
</table>

Table 3: shows the mean square error (MSE) & percentage relative efficiency (PRE) for the three populations.
Results & Discussion
Class of ratio estimators of finite population mean is proposed and performance of anticipated estimators over existing estimators were established. The scope of the study is to analyze and estimate the biasness, mean square errors of anticipated estimators and efficiency comparison with some existing estimators. Tables 2 and 3 show the results of the Constant, Bias, Mean Square Error (MSE) & Percentage Relative Efficiency (PRE) of anticipated & existing estimators considered in study for all populations used. Outcomes also discovered that anticipated estimators have least MSE and advanced PRE than other estimators. The outcomes also show that average dispersion of anticipated estimators gives better estimates on the average compare to other estimators considered.

Future Scope
The future scope of study is to transform the sampling technique from simple random sampling to other sampling techniques like stratified sampling, two stage sampling, cluster sampling or successive sampling.

Conclusion
In Table 3, anticipated estimators performed better than prevailing estimators considered in study. So, it is clear that anticipated estimators performed superior than other estimators having minimum Mean Square Error (MSE) & highest Percentage Relative Error (PRE). We therefore conclude that anticipated estimators are relatively efficient and better than other estimators for estimation of population mean.

Acknowledgements
The authors are grateful to Kebbi State University of Science and Technology Aliero, Nigeria for providing the resources in conducting this research.

Funding
The authors received no financial support for the research, authorship, and/or publication of this article.

Conflict of Interest
The authors declare no conflict of interest.

References
<table>
<thead>
<tr>
<th></th>
<th>Authors</th>
<th>Title</th>
<th>Journal</th>
</tr>
</thead>
</table>