



Solving Linear Programming Problems with Grey Data

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Abstract

A Grey Linear Programming problem differs from an ordinary one to the fact that the coefficients of its objective function and / or the technological coefficients and constants of its constraints are grey instead of real numbers. In this work a new method is developed for solving such kind of problems by the whitenization of the grey numbers involved and the solution of the obtained in this way ordinary Linear Programming problem with a standard method. The values of the decision variables in the optimal solution may then be converted to grey numbers to facilitate a vague expression of it, but this must be strictly checked to avoid non creditable such expressions. Examples are also presented to illustrate the applicability of our method in real life applications



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Introduction

It is well known that Linear Programming (LP) is a technique for the optimization (maximization or minimization) of a linear objective function subject to linear equality and inequality constraints. The feasible region of a LP problem is a convex polytope, which is a generalization of the three-dimensional polyhedron in the n -dimensional space R^n , where R denotes the set of the real numbers and n is an integer, $n \geq 2$.

In 1975 the Soviet Leonid Kantorovich and the Dutch-American T. C. Koopmans shared the Nobel prize in

Economics for their work, at the end of the 1930's, on formulating and solving LP problems. A LP algorithm determines a point of the LP polytope, where the objective function takes its optimal value, if such a point exists. In 1947 George B. Dantzig invented the SIMPLEX algorithm¹ that for the first time efficiently tackled the LP problem in most cases. Note that earlier, in 1941, Frank Lamien Hitchcock gave a very similar to the SIMPLEX algorithm solution for the Transportation Problem. In 1948 Dantzig, adopting a conjecture of John von Neuman, who worked on an equivalent problem in Game Theory, provided a formal proof of the theory of Duality². According to

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the above theory every LP problem can be converted to a dual problem the optimal solution of which, if it exists, provides an optimal solution for the original problem. A large breakthrough in the field came also in 1984, when Narendra Karmakar introduced a new interior-point method for solving LP problems. For more details about the history of LP the reader may look at³, while for general facts about the SIMPLEX algorithm we refer to Chapters 3 and 4 of⁴.

LP, apart from mathematics, is widely used nowadays in business and economics, in several engineering problems, etc. Many practical problems of Operations Research can be expressed as LP problems. However, in large and complex systems, like the socio-economic, the biological ones, etc., it is often very difficult to solve satisfactorily the LP problems with the standard theory, since the necessary data can not be easily determined precisely and therefore estimates of them are used in practice. The reason for this is that such kind of systems they usually involve many different and constantly changing factors the relationships among which are indeterminate, making their operation mechanisms not clear. In order to obtain good results in such cases one may apply either techniques of fuzzy logic (*Fuzzy LP*, e.g. see⁵⁻⁷, etc.) or of the grey systems theory (*Grey LP*, e.g. see^{8,9}, etc.).

In this work we develop a new technique for solving grey LP problems. The rest of the paper is formulated as follows: In the second Section the background information is recalled about the *Grey Numbers (GNs)* which is necessary for the understanding of the paper. In the third Section our method for solving grey LP problems is developed and examples are presented illustrating it. Finally, the fourth Section is devoted to our conclusions and some hints for future research on the subject.

Grey Numbers

An investigated object with "poor" information is referred to as a grey system. An effective tool for handling the approximate data of a grey system is the use of GNs. In an earlier work we have presented the GNs and their arithmetic and we have applied them for assessing the mean performance of a group of objects (students, players, etc.) participating in a certain activity¹⁰. Therefore, here we shall recall only

the information about GNs which is necessary for the understanding of the present work.

A GN is an indeterminate number whose probable range is known, but which has unknown position within its boundaries. Therefore, a GN, say A , can be expressed mathematically in the form

$$A \in [a, b] = \{x \in R: a \leq x \leq b\},$$

where R is the set of the real numbers. If $a = b$, then A is called white number and if $A \in (-\infty, +\infty)$, then A is called black number.

Let us denote by $w(A)$ the white number with the highest probability to be the representative value of the GN $A [a, b]$. The technique of determining the value of $w(A)$ is called *whitenization* of A .

One usually defines $w(A) = (1-t)a + tb$, with t in $[0, 1]$. This is known as the equal weight *whitenization*. When the distribution of A is unknown, one takes $t = 1/2$, which gives that $w(A) = \frac{a+b}{2}$.

For general facts on GNs we refer to the book¹¹. Further, for the needs of the present paper we introduce the following definition:

The GN $A [a, b]$ has *Rank of Greyness (RoG)* equal to x , with x in R , if, and only if $b - a = x$. We write then $\text{RoG}(A) = x$. Therefore, for a white number A we have $\text{RoG}(A) = 0$, whereas the RoG of a black number is defined to be equal to $+\infty$.

Note that, if the values $w(A) = y$ and $\text{RoG}(A) = x$ are known, then the GN $A \in [a, b]$ can be easily determined by solving the system of equations $y = (1-t)a + tb$ and $b - a = x$. For example, if $w(A) = 4$ and $\text{RoG}(A) = 2$, then $a + b = 8$ and $b - a = 2$, which gives that $a = 3$ and $b = 5$.

A Method of Solving Grey LP Problems

The general form of a Grey LP problem is the following:

Maximize (or minimize) the linear expression $F = A_1x_1 + A_2x_2 + \dots + A_nx_n$ subject to constraints of the form $x_j \geq 0$, $A_{11}x_1 + A_{12}x_2 + \dots + A_{1n}x_n \leq (\geq) B_1$, where $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ and A_{ij}, A_j, B_i are GNs.

The proposed in this work method for solving a Grey LP problem involves the following steps:

- Whitenization of the GNs A_j, A_{ij} and B_i .
- Solution of the obtained by the previous step ordinary LP problem with the standard theory.
- Conversion of the values of the decision variables in the optimal solution to GNs with the desired RoG.

The last step is not compulsory, but it is useful in problems of fuzzy structure, where a vague expression of their solution is often preferable than the crisp one.

The following examples illustrate the applicability of our method: in real life applications:

Example 1: A furniture-making factory constructs tables and desks. It has been statistically estimated that the construction of a group of tables needs 2-3 working hours (w.h.) for assembling, 2.5-3.5 w.h. for elaboration (plane, etc.) and 0.75-1.25 w.h. for polishing. On the other hand, the construction of a group of desks needs 0.8-1.2, 2-4 and 1.5-2.5 w.h. for each of the above procedures respectively. According to the factory's existing number of workers, no more than 20 w.h. per day can be spent for the assembling, no more than 30 w.h. for the elaboration and no more than 18 w.h. for the polishing of the tables and desks. If the profit from the sale of a group of tables is between 2.7 and 3.3 thousand euros and of a group of desks between 3.8 and 4.2 thousand euros, find how many groups of tables and desks should be constructed daily to maximize the factory's total profit. Express the problem's optimal solution with GNs of RoG equal to 1.

Solution: Let x_1 and x_2 be the groups of tables and desks to be constructed daily. Then the problem is mathematically formulated as follows:

Maximize $F = [2.7, 3.3]x_1 + [3.8, 4.2]x_2$ subject to constraints $x_1, x_2 \geq 0$ and
 $[2, 3]x_1 + [0.8, 1.2]x_2 \leq [20, 20]$
 $[2.5, 3.5]x_1 + [2, 4]x_2 \leq [30, 30]$
 $[0.75, 1.25]x_1 + [1.5, 2.5]x_2 \leq [16, 16]$

The whitenization of the GNs involved leads to the following LP maximization problem of canonical form:

Maximize $f(x_1, x_2) = 3x_1 + 4x_2$ subject to the constraints $x_1, x_2 \geq 0$ and
 $2.5x_1 + x_2 \leq 20$
 $3x_1 + 3x_2 \leq 30$
 $x_1 + 2x_2 \leq 16$

Adding the slack variables s_1, s_2, s_3 for converting the last three inequalities to equations one forms the problem's first SIMPLEX matrix, which corresponds to the feasible solution $f(0, 0) = 0$, as follows:

$$\left[\begin{array}{cccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & & \text{Const.} \\ - & - & - & - & - & - & - \\ 2.5 & 1 & 1 & 0 & 0 & | & 20 = s_1 \\ 3 & 3 & 0 & 1 & 0 & | & 30 = s_2 \\ 1 & 2 & 0 & 0 & 1 & | & 16 = s_3 \\ - & - & - & - & - & | & - \\ -3 & -4 & 0 & 0 & 0 & | & 0 = f(0,0) \end{array} \right]$$

Denote by L_1, L_2, L_3, L_4 the rows of the above matrix, the fourth one being the net evaluation row. Since -4 is the smaller (negative) number of the net evaluation row and $16/2 < 30/3 < 20/1$, the pivot element 2 lies in the intersection of the third row and second column. Therefore, applying the linear transformations $L_3 \rightarrow 1/2 L_3 = L'_3$ and $L_1 \rightarrow L_1 - L'_3, L_2 \rightarrow L_2 - 3L'_3, L_4 \rightarrow L_4 + 4L'_3$, one obtains the second SIMPLEX matrix, which corresponds to the feasible solution $f(0, 8) = 32$ and it is the following:

$$\left[\begin{array}{cccccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & & \text{Const.} \\ - & - & - & - & - & - & - \\ 2 & 0 & 1 & 0 & -\frac{1}{2} & | & 12 = s_1 \\ \frac{3}{2} & 0 & 0 & 1 & -\frac{3}{2} & | & 6 = s_2 \\ \frac{1}{2} & 1 & 0 & 0 & \frac{1}{2} & | & 8 = x_2 \\ - & - & - & - & - & | & - \\ -1 & 0 & 0 & 0 & 0 & | & 32 = f(0,8) \end{array} \right]$$

*The profit depends upon the price of the wood, the salaries of the workers, etc.

In this matrix the pivot element $3/2$ lies in the intersection of the second row and first column, therefore working as above one obtains the third SIMPLEX matrix, which is:

Since there is no negative index in the net evaluation row, this is the last SIMPLEX matrix. Therefore $f(4, 6) = 36$ is the optimal solution maximizing the objective function. Further, since both the decision variables x_1 and x_2 are basic variables, i.e. they participate in the optimal solution, the above solution is unique.

$$\left[\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & s_3 & \text{Const.} \\ - & - & - & - & - & - \\ 0 & 0 & 1 & -\frac{4}{3} & -\frac{3}{2} & 4 = s_1 \\ 1 & 0 & 0 & \frac{2}{3} & -1 & 4 = x_1 \\ 0 & 1 & 0 & -\frac{1}{3} & 1 & 6 = x_2 \\ - & - & - & - & - & - \\ 0 & 0 & 0 & \frac{2}{3} & 1 & 36 = f(4,6) \end{array} \right]$$

Converting the values of the decision variables in the above solution to GNs with RoG equal to 1 one finds that

$$x_1 \in [3.5, 4.5] \text{ and } x_2 \in [5.5, 6.5].$$

Therefore a vague expression of the solution states that the factory's maximal profit corresponds to a daily production between 3.5 and 4.5 groups of tables and between 5.5 and 6.5 groups of desks.

However, considering for example the extreme values of the daily construction of 4.5 groups of tables and 6.5 groups of desks, one finds that it needs 33 in total w.h. for elaboration, whereas the maximum available w.h. are only 30. In other words, a vague expression of the solution does not guarantee that all the values of the decision variables within the boundaries of the corresponding GNs are feasible solutions.

Example 2: Three kinds of food, say T_1 , T_2 and T_3 , are used in a poultry farm for feeding the chickens, their cost varying between 38 - 42, 17 - 23 and 55 -

65 cents per kilo respectively. The food T_1 contains 1.5 - 2.5 units of iron and 4 - 6 units of vitamins per kilo, T_2 contains 3.2 - 4.8, 0.6 - 1.4 and T_3 contains 1.7 - 2.3, 0.8 - 1.2 units per kilo respectively. It has been decided that the chickens must receive at least 24 units of iron and 8 units of vitamins per day. How one must mix the three foods so that to minimize the cost of the food? Express the problem's solution with GNs of RoG equal to 2.

Solution: Let x_1 , x_2 and x_3 be the quantities in kilos for each food to be mixed. Then, the problem's mathematical model is the following:

Minimize
 $F = [38, 42]x_1 + [17, 23]x_2 + [55, 65]x_3$ subject to the constraints $x_1, x_2, x_3 \geq 0$
 $[1.5, 2.5]x_1 + [3.2, 4.8]x_2 + [1.7, 2.3]x_3 \geq [24, 24]$
 $[4, 6]x_1 + [0.6, 1.4]x_2 + [0.8, 1.2]x_3 \geq [8, 8]$.

The whitenization of the GNs leads to the following LP minimization problem of canonical form:

Minimize $f(x_1, x_2, x_3) = 40x_1 + 20x_2 + 60x_3$ subject to the constraints $x_1, x_2, x_3 \geq 0$ and
 $2x_1 + 4x_2 + 2x_3 \geq 24$
 $5x_1 + x_2 + x_3 \geq 8$

The dual of the above problem is: the following:
 Maximize $g(z_1, z_2) = 24z_1 + 8z_2$ subject to the constraints $z_1, z_2 \geq 0$ and
 $2z_1 + 5z_2 \leq 40$
 $4z_1 + z_2 \leq 20$
 $2z_1 + z_2 \leq 60$

Working similarly with Example 1 it is straightforward to check that the last SIMPLEX matrix of the dual problem is the following:

$$\left[\begin{array}{cccc|c} z_1 & z_2 & s_1 & s_2 & s_3 & \text{Const.} \\ - & - & - & - & - & - \\ 0 & 1 & \frac{2}{9} & \frac{1}{9} & 0 & \frac{20}{3} = z_2 \\ 1 & 0 & -\frac{1}{18} & \frac{5}{18} & 0 & \frac{10}{3} = z_1 \\ 0 & 0 & -\frac{1}{9} & -\frac{4}{9} & 1 & \frac{140}{3} = s_3 \\ - & - & - & - & - & - \\ 0 & 0 & \frac{4}{9} & \frac{52}{9} & 0 & \frac{400}{3} = g\left(\frac{10}{3}, \frac{20}{3}\right) \end{array} \right]$$

Therefore the solution of the original minimization problem is $f_{\min} = f(4/9, 52/9, 0) = 400/3$. In other words, the minimal cost of the chickens' food is $400/3 \approx 133$ cents and will be succeeded by mixing $4/9 \approx 0.44$ kilos from food T_1 and $52/9 \approx 5.77$ kilos from food T_2 .

Converting the values of the decision variables in the above solution to GNs with RoG equal to 2 one finds that $x_1 \in [-5/9, 13/9]$, $x_2 \in [43/9, 61/9]$ and $x_3 \in [-1, 1]$. However, since $x_1, x_2, x_3 \geq 0$, one must finally take $x_1 \in [0, 13/9]$ and $x_3 \in [0, 1]$.

Therefore a vague expression of the optimal solution states that the minimal cost of the chickens' food will be succeeded by mixing at most $13/9 \approx 1.44$, between $43/9 \approx 4.77$ and $61/9 \approx 6.77$ and at most 1 kilos from each one of the foods T_1, T_2 and T_3 respectively.

However, considering the extreme values $x_1 = 13/9$, $x_2 = 61/9$ and $x_3 = 1$, one finds that the cost of the food becomes equal to $13/9 * 40 + 61/9 * 20 + 60 \approx 253$ cents, which exceeds too much the minimal price of 133 cents. This means that the chosen RoG of the GNs is too large to provide a creditable vague expression of the problem's solution. On the contrary, choosing for example the RoG to be equal to 0.5, one finds that $x_1 \in [5/36, 23/36]$, $x_2 \in [199/36, 217/36]$ and $x_3 \in [-1/4, 1/4]$. Then $23/36 * 40 + 217/36 * 20 + 1/4 * 60 \approx 141$ cents, which is much closer to the minimal price of 133 cents. In general, the smaller is the chosen RoG of the GNs the more creditable is the vague expression of the problem's solution.

Table 1: Required quantities of milk

| | T_1 | T_2 | T_3 |
|---|------------|------------|------------|
| C | [1, 3] | [5, 7] | [0.5, 1.5] |
| S | [3, 5] | [2, 4] | [0.5, 1.5] |
| P | [0.8, 1.2] | [0.3, 0.7] | [0.2, 0.8] |

Example 3: A cheese-making company produces three different types of cheese T_1, T_2 and T_3 by mixing cow-milk (C), sheep-milk (S) and milk powder (P).

The required quantities in kilos from each kind of milk for producing a barrel of each of the three types of cheese are depicted, in form of GNs, in the following Table:

The cheese-maker's profit from the sale of a barrel of cheese is 3 thousand euros for T_1 , 2 thousand euros for T_3 , whereas from the sale of a barrel of T_2 , the production of which becomes necessary for marketing reasons, there is a loss of 1 thousand euros.

At the end of a certain day the stock of the cow-milk is high, so that at least 200 kilos of it must be used the next day, whereas the stock of the sheep-milk is 150 kilos. Further, there exists a stock of 100 kilos of expiring milk powder all of which must be spent the next day. Under the above conditions find with RoG equal to 0.2 which must be the next day's production of cheese in order to maximize the profit from its sale.

Solution: Let x_1, x_2 and x_3 be the barrels to be produced of cheese of the types T_1, T_2 and T_3 respectively. Then the problem is mathematically formulated as follows:

Maximize
 $F = [3,3]x_1 - [1,1]x_2 + [2, 2]x_3$ subject to the constraints
 $x_1, x_2, x_3 \geq 0$ and
 $[1,3]x_1 + [5,7]x_2 + [0.5,1.5]x_3 \geq [200,200]$,
 $[3,5]x_1 + [2,4]x_2 + [1.5,2.5]x_3 \geq [150,150]$
 $[1.8,2.2]x_1 + [0.8,1.2]x_2 + [0.6,1.4]x_3 = [100,100]$.

The whitenization of the GNs leads to the following LP maximization problem of general form †:

Maximize $f(x_1, x_2, x_3) = 3x_1 - x_2 + 2x_3$ subject to the constraints $x_1, x_2, x_3 \geq 0$ and
 $2x_1 + 6x_2 + x_3 \geq 200$
 $4x_1 + 3x_2 + 2x_3 \geq 150$
 $2x_1 + x_2 + x_3 - 0.020 = 100$.

Adding the surplus variable s_1 to the first inequality, the slack variable s_2 to the second one and the artificial variables t_1 and t_2 to the first inequality

*The grey data of Table 1 show that the production of a barrel of T_1 requires quantities of cow-milk between 2 and 6 kilos and of sheep-milk and milk powder between 1 and 3 kilos, etc.

*It is recalled that a LP problem of general form differs from a problem of canonical form to the fact that there exists at least a constraint having the inverse sign of inequality (here $=$), whereas there could also exist constraints with the sign of equality

and the last equation one turns all the special constraints to equations. Next, adding by members the two equations containing the artificial variables, one forms the problem's first *generalized SIMPLEX matrix* as follows:

$$\left[\begin{array}{cccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & t_1 & t_2 & \text{Const.} \\ - & - & - & - & - & - & - & - \\ 2 & 6 & 1 & -1 & 0 & 1 & 0 & 200 = t_1 \\ 4 & 3 & 2 & 0 & 1 & 0 & 0 & 150 = s_2 \\ 2 & 1 & 3 & 0 & 0 & 0 & 1 & 100 = t_2 \\ - & - & - & - & - & - & - & - \\ -3 & 1 & -2 & 0 & 0 & 0 & 0 & 0 = f(0,0,0) \\ - & - & - & - & - & - & - & - \\ 4 & 7 & 4 & -1 & 0 & 1 & 1 & t_1 + t_2 = 300 \end{array} \right]$$

The rows of the artificial variables t_1 and t_2 are the, so called, anonymous rows of the above matrix. For the pivoting process, considering all the columns containing at least one positive number in the anonymous rows, we choose the one having the greatest positive number in the last row (of $t_1 + t_2$), i.e. the column of x_2 . Then, since $200/6 < 150/3 < 100/1$, the pivot element 6 lies in the first row. Therefore, applying the proper linear transformations among the rows of the matrix one forms the second *generalized SIMPLEX matrix* as follows:

$$\left[\begin{array}{cccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & t_1 & t_2 & \text{Const.} \\ - & - & - & - & - & - & - & - \\ \frac{1}{3} & 1 & \frac{1}{6} & -\frac{1}{6} & 0 & \frac{1}{6} & 0 & \frac{100}{3} = x_2 \\ 3 & 0 & \frac{3}{2} & \frac{1}{2} & 1 & -\frac{1}{2} & 0 & 50 = s_2 \\ \frac{5}{3} & 0 & \frac{17}{6} & \frac{1}{6} & 0 & \frac{1}{6} & 1 & \frac{200}{3} = t_2 \\ - & - & - & - & - & - & - & - \\ -\frac{10}{3} & 0 & -\frac{13}{6} & \frac{1}{6} & 0 & -\frac{1}{6} & 0 & -\frac{100}{3} = f(0, \frac{100}{3}, 0) \\ - & - & - & - & - & - & - & - \\ \frac{5}{3} & 0 & \frac{17}{6} & \frac{1}{6} & 0 & -\frac{1}{6} & 1 & \frac{200}{3} = t_1 + t_2 \end{array} \right]$$

The pivot element $17/6$ lies now in the intersection of the column of x_3 and the row of t_2 and the third *generalized SIMPLEX matrix* is the following:

Therefore, omitting the last row and the columns of the artificial variables one obtains the problem's first *canonical SIMPLEX matrix*.

Next, continuing the process in the standard way one finally reaches the optimal solution

$$f_{\max} = f\left(\frac{125}{18}, \frac{250}{9}, \frac{175}{9}\right) = 575/18 .$$

$$\left[\begin{array}{cccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & t_1 & t_2 & \text{Const.} \\ - & - & - & - & - & - & - & - \\ \frac{4}{17} & 1 & 0 & \frac{3}{17} & 0 & \frac{3}{17} & -\frac{1}{17} & \frac{500}{17} = x_2 \\ \frac{36}{17} & 0 & 0 & \frac{7}{17} & 1 & -\frac{7}{17} & -\frac{9}{17} & \frac{250}{17} = s_2 \\ \frac{10}{17} & 0 & 1 & \frac{1}{17} & 0 & -\frac{1}{17} & -\frac{6}{17} & \frac{400}{17} = x_3 \\ - & - & - & - & - & - & - & - \\ \frac{35}{17} & 0 & 0 & \frac{5}{17} & 0 & \frac{5}{17} & \frac{13}{17} & \frac{300}{17} = f(0, \frac{500}{17}, \frac{400}{17}) \\ - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 = t_1 + t_2 \end{array} \right]$$

Converting the optimal values of the decision variables to GNs with RoG equal to 0.2 one finds that $x_1 \in [6.84, 7.04]$, $x_2 \in [27.68, 27.88]$, $x_3 \in [19.34, 19.54]$.

Therefore a vague expression of the problem's optimal solution states that, if the next day's production is between 6.84-7.04, 27.68-27.88 and 19.34-19.54 barrels of the cheese T_1 , T_2 and T_3 respectively, then the profit from their sale will be maximized to around $575/18 \approx 31.9444$ thousand euros.

Conclusion

A new technique was developed in this paper for solving Grey LP problems by the whitenization of the GNs involved in their statements and the solution of the ordinary LP problem obtained in this way with the standard theory. Real life examples were also presented to illustrate our method. In LP problems with fuzzy structure a vague expression of their solution is often preferable than the crisp one. This was attempted in the present work by converting the values of the decision variables in the optimal solution of the corresponding ordinary LP problem to GNs with the desired RoG. The smaller is the value of the chosen RoG, the more creditable the vague expression of the problem's optimal solution.

An analogous method could be applied for solving fuzzy LP problems and systems of equations with grey or fuzzy data and this is the main target of our future research on the subject.

References

1. Dantzig, G.B., Maximization of a linear function of variables subject to linear inequalities, 1947. Published in: T.C. Koopmans (Ed.): *Activity Analysis of Production and Allocation*, pp. 339–347. Wiley & Chapman - Hall, New York, London, 1951.
2. Dantzig, G.B., Reminiscences about the origins of linear programming, Technical Report of Systems Optimization Laboratory 81-5, Department of Operations Research, Stanford University, CA, April 1981.
3. Wikipedia, Linear Programming, retrieved from https://en.wikipedia.org/wiki/Linear_programming on March 2018.
4. Taha, H.A., *Operations Research – An Introduction*, Second Edition, Collier Macmillan, N.Y., London, 1967.
5. Tanaka H., Asai K., Fuzzy linear programming problems with fuzzy numbers, *Fuzzy Sets and Systems*, 13, 1-10, 1984.
6. Verdegay, J.L., A dual approach to solve the fuzzy linear programming problem, *Fuzzy Sets and Systems*, 14, 131-141, 1984.
7. Stephen Dinagar, D., Kamalanathan, S., Solving Linear Programming Problem Using New Ranking Procedures of Fuzzy Numbers, *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, 281-292, 2017.
8. Huang, G., Dan Moore, R., Grey linear programming, its solving approach, and its application, *International Journal of Systems Science*, 24(1), 159-172, 1993
9. Seyed H., Razavi H., Akrami, H., Hashemi, S.S., A multi objective programming approach to solve grey linear programming, *Grey Systems: Theory and Application*, 2(2), 259-271, 2012.
10. Voskoglou, M. Gr., Theodorou Y., Application of Grey Numbers to Assessment processes, *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, 7, 273-280, 2017.
11. Liu, S. F. & Lin, Y. (Eds.), *Advances in Grey System Research*, Springer, Berlin – Heidelberg, 2010.